Chapter 1: Differentiation

EXERCISE 1.1 [PAGES 11 - 13]

Exercise 1.1 | Q 1.1 | Page 11

Differentiate the following w.r.t.x: $(x^3 - 2x - 1)^5$

SOLUTION

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Method 1:
Let y = (x^3 - 2x - 1)^5
Put u = x^3 - 2x - 1. Then y = u^5
\therefore \frac{\mathrm{d}y}{\mathrm{d}u} = \frac{\mathrm{d}}{\mathrm{d}u} \left( u^5 \right) = 5 \mathrm{u}^4
= 5(x3 - 2x - 1)4
and
\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}\mathbf{x}} = \frac{\mathrm{d}}{\mathrm{d}\mathbf{x}}(x^3 - 2x - 1)
 = 3x2 - 2 x 1 - 0
= 3x^2 - 2
\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{dy}}{\mathrm{du}} \times \frac{\mathrm{du}}{\mathrm{dx}}
= 5(x^3 - 2x - 1)^4(3x^2 - 2)
= 5(3x^2 - 2)(x^3 - 2x - 1)^4.
Method 2 :
Let y = (x^3 - 2x - 1)^5
Differentiating w.r.t. x, we get
\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}(x^3 - 2x - 1)^5
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$$= 5(x^{3}-2x-1)4 \times \frac{d}{dx}(x^{3}-2x-1)$$
$$= 5(x^{3}-2x-1)^{4} \times (3x^{2}-2 \times 1-0)$$
$$= 5(3x^{2}-2)(x^{3}-2x-1)^{4}.$$

Note : Out of the two methods given above, we will use Method 2 for solving the remaining problems.

Exercise 1.1 | Q 1.2 | Page 11

Differentiate the following w.r.t.x: $\left(2x^{rac{3}{2}}-3x^{rac{4}{3}}-5
ight)^{rac{5}{2}}$

SOLUTION

Let
$$y = \left(2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5\right)^{\frac{5}{2}}$$

Differentiating w.r.t. x,we get
 $\frac{dy}{dx} = \frac{d}{dx} \left(2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5\right)^{\frac{5}{2}}$
 $= \frac{5}{2} \left(2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5\right)^{\frac{5}{2}-1} \times \frac{d}{dx} \left(2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5\right)$
 $= \frac{5}{2} \left(2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5\right)^{\frac{3}{2}} \times \left(2 \times \frac{3}{2}x^{\frac{3}{2}-1} - 3 \times \frac{4}{3}x^{\frac{4}{3}-1} - 0\right)$
 $= \frac{5}{2} \left(2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5\right)^{\frac{3}{2}} \left(3x^{\frac{1}{2}} - 4x^{\frac{1}{3}}\right)$
 $= \frac{5}{2} \left(3\sqrt{x} - 4\sqrt[3]{x}\right) \left(2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5\right)^{\frac{3}{2}}.$

Exercise 1.1 | Q 1.3 | Page 11

Differentiate the following w.r.t.x: $\sqrt{x^2+4x-7}$



$$y = \sqrt{x^{2} + 4x - 7} \left[\sqrt{x} = \frac{1}{2\sqrt{x}} \right]$$

Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{1}{2\sqrt{x^{2} + 4x - 7}} \cdot \frac{d}{dx} \left(x^{2} + 4x - 7\right)$$
$$= \frac{1}{2\sqrt{x^{2} + 4x - 7}} \left(\frac{d}{dx} x^{2} + \frac{d}{dx} 4x - \frac{d}{dx} 7 \right)$$
$$= \frac{1}{2\sqrt{x^{2} + 4x - 7}} \cdot (2x + 4 - 0)$$
$$= \frac{2(x + 2)}{2\sqrt{x^{2} + 4x - 7}}$$
$$= \frac{(x + 2)}{\sqrt{x^{2} + 4x - 7}}.$$

Exercise 1.1 | Q 1.4 | Page 11

Differentiate the following w.r.t.x: $\sqrt{x^2+\sqrt{x^2+1}}$

Let
$$y = \sqrt{x^2 + \sqrt{x^2 + 1}}$$

Differentiating w.r.t. x,we get,
 $\frac{dy}{dx} = \frac{d}{dx} \left(x^2 + \sqrt{x^2 + 1}\right)^{\frac{1}{2}}$
 $= \frac{1}{2} \left(x^2 + \sqrt{x^2 + 1}\right)^{-\frac{1}{2}} \cdot \frac{d}{dx} \left(x^2 + \sqrt{x^2 + 1}\right)$
 $= \frac{1}{2\sqrt{x^2 + \sqrt{x^2 + 1}}} \cdot \left[\frac{d}{dx} \left(x^2\right) + \frac{d}{dx} \left(\sqrt{x^2 + 1}\right)\right]$
 $= \frac{1}{2\sqrt{x^2 + \sqrt{x^2 + 1}}} \cdot \left[2x + \frac{1}{2\sqrt{x^2 + 1}} \cdot \frac{d}{dx} \left(x^2 + 1\right)\right]$





$$= \frac{1}{2\sqrt{x^2 + \sqrt{x^2 + 1}}} \cdot \left[2x + \frac{1}{2\sqrt{x^2 + 1}} (2x + 0) \right]$$
$$= \frac{1}{2\sqrt{x^2 + \sqrt{x^2 + 1}}} \cdot \left[2x + \frac{x}{\sqrt{x^2 + 1}} \right]$$
$$= \frac{1}{2\sqrt{x^2 + \sqrt{x^2 + 1}}} \cdot \left[\frac{2x\sqrt{x^2 + 1 + x}}{\sqrt{x^2 + 1}} \right]$$
$$= \frac{x\left(2\sqrt{x^2 + 1} + 1\right)}{2\sqrt{x^2 + 1}} \cdot \sqrt{x^2 + \sqrt{x^2 + 1}} \cdot$$

Exercise 1.1 | Q 1.5 | Page 11

Differentiate the following w.r.t.x: $rac{3}{5\sqrt[3]{(2x^2-7x-5)^5}}$

SOLUTION

Let
$$y = \frac{3}{5\sqrt[3]{(2x^2 - 7x - 5)^5}}$$

Differentiating w.r.t. x,we get,
 $\frac{dy}{dx} = \frac{3d}{5dx}(2x^2 - 7x - 5)^{-\frac{5}{3}}$
 $= \frac{3}{5} \times (-\frac{5}{3})(2x^2 - 7x - 5)^{-\frac{5}{3} - 1} \cdot \frac{d}{dx}(2x^2 - 7x - 5)$
 $= -(2x^2 - 7x - 5)^{-\frac{8}{3}} \cdot (2 \times 2x - 7 \times 1 - 0)$
 $= -\frac{4x - 7}{(2x^2 - 7x - 5)^{\frac{8}{3}}}$.

Exercise 1.1 | Q 1.6 | Page 11

Differentiate the following w.r.t.x:
$$\left(\sqrt{3x-5}-rac{1}{\sqrt{3x-5}}
ight)^5$$

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Let
$$y = \left(\sqrt{3x-5} - \frac{1}{\sqrt{3x-5}}\right)^5$$

Differentiating w.r.t. xwe get,
 $\frac{dy}{dx} = \frac{d}{dx} \left(\sqrt{3x-5} - \frac{1}{\sqrt{3x-5}}\right)^5$
 $= 5\left(\sqrt{3x-5} - \frac{1}{\sqrt{3x-5}}\right)^4 \cdot \left[\frac{d}{dx}\left(\sqrt{3x-5} - \frac{1}{\sqrt{3x-5}}\right)\right]$
 $= 5\left(\sqrt{3x-5} - \frac{1}{\sqrt{3x-5}}\right)^4 \cdot \left[\frac{1}{2}(3x-5)^{-\frac{1}{2}} \cdot \frac{d}{dx}(3x-5) - \left(-\frac{1}{2}\right)(3x-5)^{-\frac{x}{2}} \cdot \frac{d}{dx}(3x-5)\right]$
 $= 5\left(\sqrt{3x-5} - \frac{1}{\sqrt{3x-5}}\right)^4 \times \left[\frac{1}{2}(3x-5)^{-\frac{1}{2}} \cdot (3 \times 1 - 0) + \frac{1}{2(3x-5)^{\frac{x}{2}}} \cdot (3 \times 1 - 0)\right]$
 $= \frac{15}{2}\left(\sqrt{3x-5} - \frac{1}{\sqrt{3x-5}}\right)^4 \cdot \left[\frac{3}{2\sqrt{3x-5}} + \frac{3}{2(3x-5)^{\frac{x}{2}}}\right]$
 $= \frac{15(3x-4)}{2(3x-5)^{\frac{3}{2}}}\left(\sqrt{3x-5} - \frac{1}{\sqrt{3x-5}}\right)^4 \cdot \left[\frac{3x-5+1}{(3x-5)^{\frac{3}{2}}}\right]$

Exercise 1.1 | Q 2.01 | Page 12

Differentiate the following w.r.t.x: $\cos(x^2 + a^2)$

SOLUTION





Let
$$y = \cos(x^2 + a^2)$$

Differentiating w.r.t. x,we get,

$$\frac{dy}{dx} = \frac{d}{dx} \left[\cos(x^2 + a^2) \right]$$

$$= -\sin(x^2 + a^2) \cdot \frac{d}{dx} (x^2 + a^2)$$

$$= -\sin(x^2 + a^2) \cdot (2x + 0)$$

$$= -2x \sin(x^2 + a^2) \cdot (2x + 0)$$
Exercise 1.1 | Q 2.02 | Page 12

Differentiate the following w.r.t.x: $\sqrt{e^{(3x+2)+5}}$

SOLUTION

Let
$$y = \sqrt{e^{(3x+2)+5}}$$

Differentiating w.r.t. x,we get,
 $\frac{dy}{dx} = \frac{d}{dx} \left[e^{(3x+2)} + 5 \right]^{\frac{1}{2}}$
 $= \frac{1}{2} \left[e^{(3x+2)} + 5 \right]^{-\frac{1}{2}} \cdot \frac{d}{dx} \left[e^{(3x+2)} + 5 \right]$
 $= \frac{1}{2\sqrt{e^{(3x+2)}+5}} \cdot \left[e^{(3x+2)} \cdot \frac{d}{dx} (3x+2) + 0 \right]$
 $= \frac{1}{2\sqrt{e^{(3x+2)}+5}} \cdot \left[e^{(3x+2)} \cdot (3 \times 1 + 0) \right]$
 $= \frac{3e^{(3x+2)}}{2\sqrt{e^{(3x+2)+5}}} \cdot$

Exercise 1.1 | Q 2.03 | Page 12

Differentiate the following w.r.t.x: $\log \left[tan \left(rac{x}{2}
ight)
ight]$

SOLUTION



Let
$$y = \log \left[\tan \left(\frac{x}{2} \right) \right]$$

Differentiating w.r.t. x, we get,
 $\frac{dy}{dx} = \frac{d}{dx} \log \left[\tan \left(\frac{x}{2} \right) \right]$
 $= \frac{1}{\tan \left(\frac{x}{2} \right)} \cdot \frac{d}{dx} \left[\tan \left(\frac{x}{2} \right) \right]$
 $= \frac{1}{\tan \left(\frac{x}{2} \right)} \cdot \sec^2 \left(\frac{x}{2} \right) \cdot \frac{d}{dx} \left(\frac{x}{2} \right)$
 $= \frac{\cos \left(\frac{x}{2} \right)}{\sin \left(\frac{x}{2} \right)} \cdot \frac{1}{\cos^2 \left(\frac{x}{2} \right)} \cdot \frac{1}{2} \times 1$
 $= \frac{1}{2 \sin \left(\frac{x}{2} \right) \cos \left(\frac{x}{2} \right)}$
 $= \frac{1}{\sin x}$
 $= \cos c x.$

Exercise 1.1 | Q 2.04 | Page 12

Differentiate the following w.r.t.x: $\sqrt{ an \sqrt{x}}$





Let
$$y = \sqrt{\tan \sqrt{x}}$$

Differentiating w.r.t. x,we get,
 $\frac{dy}{dx} = \frac{d}{dx} \left(\sqrt{\tan \sqrt{x}} \right)$
 $= \frac{1}{2\sqrt{\tan \sqrt{x}}} \cdot \frac{d}{dx} (\tan \sqrt{x})$
 $= \frac{1}{2\sqrt{\tan \sqrt{x}}} \times \sec^2 \sqrt{x} \cdot \frac{d}{dx} (\sqrt{x})$
 $= \frac{1}{2\sqrt{\tan \sqrt{x}}} \times \sec^2 \sqrt{x} \times \frac{1}{2\sqrt{x}}$
 $= \frac{\sec^2 \sqrt{x}}{4\sqrt{x}\sqrt{\tan \sqrt{x}}}$.

Exercise 1.1 | Q 2.05 | Page 12

Differentiate the following w.r.t.x: cot³[log(x³)]





Let
$$y = \cot^{3}[\log(x^{3})]$$

Differentiating w.r.t. x,we get,
 $\frac{dy}{dx} = \frac{d}{dx} [\cot(\log x^{3})]^{3}$
 $= 3[\cot(\log x^{3})]^{2} \cdot \frac{d}{dx} [\cot(\log x^{3})]$
 $= 3\cot^{2}[\log(x^{3})] \cdot [-\csc^{2}(\log x^{3})] \cdot \frac{d}{dx} (\log x^{3})$
 $= -3\cot^{2}[\log(x^{3})] \cdot \csc^{2}[\log(x^{3})] \cdot 3\frac{d}{dx} (\log x)$
 $= -3\cot^{2}[\log(x^{3})] \cdot \csc^{2}[\log(x^{3})] \cdot 3 \times \frac{1}{x}$
 $= \frac{-9\csc^{2}[\log(x^{3})] \cdot \cot^{2}[\log(x^{3})]}{x}$

Exercise 1.1 | Q 2.06 | Page 12

Differentiate the following w.r.t.x: $5^{\sin^3 x+3}$

SOLUTION

Let
$$y = 5^{\sin^3 x + 3}$$

Differentiating w.r.t. x,we get,
 $\frac{dy}{dx} = \frac{d}{dx} \left(5^{\sin^3 x + 3} \right)$
 $= 5^{\sin^3 x + 3} \cdot \log 5 \cdot \frac{d}{dx} (\sin^3 x + 3)$
 $= 5^{\sin^3 x + 3} \cdot \log 5 \cdot \left[3 \sin^2 x \cdot \frac{d}{dx} (\sin x) + 0 \right]$
 $= 5^{\sin^3 x + 3} \cdot \log 5 \cdot \left[3 \sin^2 x \cos x \right]$
 $= 3 \sin^2 x \cos x \cdot 5^{\sin^3 x + 3} \cdot \log 5.$

Exercise 1.1 | Q 2.07 | Page 12

Differentiate the following w.r.t.x: $\operatorname{cosec}(\sqrt{\cos x})$

SOLUTION

Let
$$y = \operatorname{cosec}(\sqrt{\cos x})$$

Differentiating w.r.t. x,we get,
 $\frac{dy}{dx} = \frac{d}{dx} [\operatorname{cosec}(\sqrt{\cos x})]$
 $= -\operatorname{cosec}(\sqrt{\cos x}) \cdot \operatorname{cot}(\sqrt{\cos x}) \cdot \frac{d}{dx} \sqrt{\cos x}$
 $= -\operatorname{cosec}(\sqrt{\cos x}) \cdot \operatorname{cot}(\sqrt{\cos x}) \cdot \frac{1}{2\sqrt{\cos x}} \cdot \frac{d}{dx} (\cos x)$
 $= -\operatorname{cosec}(\sqrt{\cos x}) \cdot \operatorname{cot}(\sqrt{\cos x}) \cdot \frac{1}{2\sqrt{\cos x}} \cdot (-\sin x)$
 $= \frac{\sin x \cdot \operatorname{cosec}(\sqrt{\cos x}) \cdot \operatorname{cot}(\sqrt{\cos x})}{2\sqrt{\cos x}}.$

Exercise 1.1 | Q 2.08 | Page 12

Differentiate the following w.r.t.x: log[cos(x³ - 5)]

SOLUTION

Let y = log[cos(x³ - 5)]
Differentiating w.r.t. x, we get,

$$\frac{dy}{dx} = \frac{d}{dx} \{ log[cos(x^{3} - 5)] \}$$

$$= \frac{1}{cos(x^{3} - 5)} \cdot \frac{d}{dx} [cos(x^{3} - 5)]$$

$$= \frac{1}{cos(x^{3} - 5)} \cdot [-sin(x^{3} - 5)] \cdot \frac{d}{dx} (x^{3} - 5)$$

$$= -\tan(x^3 - 5) \times (3x^2 - 0)$$

= -3x²tan(x³ - 5).

Exercise 1.1 | Q 2.09 | Page 12

Differentiate the following w.r.t.x: $e^{3\sin^2 x - 2\cos^2 x}$

SOLUTION

Let
$$y = e^{3\sin^2 x - 2\cos^2 x}$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} \left[e^{3\sin^2 x - 2\cos^2 x} \right]$
 $= e^{3\sin^2 x - 2\cos^2 x} \cdot \frac{d}{dx} \left(3\sin^2 x - 2\cos^2 x \right)$
 $= e^{3\sin^2 x - 2\cos^2 x} \cdot \left[3\frac{d}{dx} (\sin x)^2 - 2\frac{d}{dx} (\cos^2 x) \right]$
 $= e^{3\sin^2 x - 2\cos^2 x} \cdot \left[3 \times 2\sin x \cdot \frac{d}{dx} (\sin x) - 2 \times 2\cos x \cdot \frac{d}{dx} (\cos x) \right]$
 $= e^{3\sin^2 x - 2\cos^2 x} \cdot \left[6\sin x \cos x - 4\cos x (-\sin x) \right]$
 $= e^{3\sin^2 x - 2\cos^2 x} \cdot (10\sin x \cos x)$
 $= 5(2\sin x \cos x) \cdot e^{3\sin^2 x - 2\cos^2 x}$

Exercise 1.1 | Q 2.1 | Page 12

Differentiate the following w.r.t.x: $\cos^{2}[\log(x^{2} + 7)]$





Let
$$y = \cos^{2}[\log(x^{2} + 7)]$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} \{\cos[\log(x^{2} + 7)]\}^{2}$
 $= 2\cos[\log(x^{2} + 7)] \cdot \frac{d}{dx} \{\cos[\log(x^{2} + 7)]\}$
 $= 2\cos[\log(x^{2} + 7)] \cdot \{-\sin[\log(x^{2} + 7)]\} \cdot \frac{d}{dx} [\log(x^{2} + 7)]$
 $= -2\sin[\log(x^{2} + 7)] \cdot \cos[\log(x^{2} + 7)] \times \frac{1}{x^{2} + 7} \cdot \frac{d}{dx} (x^{2} + 7)$
 $= -\sin[2\log(x^{2} + 7)] \times \frac{1}{x^{2} + 7} \cdot (2x + 0) \dots [\because 2\sin x \cdot \cos x = \sin 2x]$
 $= \frac{-2x \cdot \sin[2\log(x^{2} + 7)]}{x^{2} + 7} \cdot (2x + 7)$

Exercise 1.1 | Q 2.11 | Page 12 Differentiate the following w.r.t.x: tan[cos (sinx)]

SOLUTION

Let y = tan[cos (sinx)]
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx} \{ tan[cos(sin x)] \}$$

$$= sec^{2}[cos(sin x)] \cdot \frac{d}{dx}[cos(sin x)]$$

$$= sec^{2}[cos(sin x)] \cdot [-sin(sin x)] \cdot \frac{d}{dx}(sin x)$$

$$= -sec^{2}[cos(sin x)] \cdot sin (sinx) \cdot cosx.$$

Exercise 1.1 | Q 2.12 | Page 12

Differentiate the following w.r.t.x: sec[tan $(x^4 + 4)$]

SOLUTION

Let
$$y = \sec[\tan (x^4 + 4)]$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} \{ \sec[\tan(x^4 + 4)] \}$
 $= \sec[\tan(x^4 + 4)] \cdot tan[\tan(x^4 + 4)] \cdot \frac{d}{dx}[\tan(x^4 + 4)]$
 $= \sec[\tan(x^4 + 4)] \cdot tan[\tan(x^4 + 4)] \cdot \sec^2(x^4 + 4) \cdot \frac{d}{dx}(x^4 + 4)]$
 $= \sec[\tan(x^4 + 4)] \cdot tan[\tan(x^4 + 4)] \cdot \sec^2(x^4 + 4) \cdot (4x^3 + 0)]$
 $= 4x^3 \cdot \sec^2(x^4 + 4) \cdot \sec[\tan(x^4 + 4)] \cdot tan[\tan(x^4 + 4)].$
Exercise 1.1 | Q.2.13 | Page 12

Differentiate the following w.r.t.x: $e^{\log[(\log x)^2 - \log x^2]}$ SOLUTION

Let
$$y = e^{\log[(\log x)^2 - \log x^2]}$$

= $(\log x)^2 - \log x^2$...[$\because e^{\log x} = x$]
= $(\log x)^2 - 2\log x$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} [\log x)^2 - 2\log x$
= $\frac{d}{dx} (\log x)^2 - 2\frac{d}{dx} (\log x)$
= $2\log x \cdot \frac{d}{dx} (\log x) - 2 \times \frac{1}{x}$



$$= 2\log x \times \frac{1}{x} - \frac{2}{x}$$
$$= \frac{2\log x}{x} - \frac{2}{x}.$$

Exercise 1.1 | Q 2.14 | Page 12

Differentiate the following w.r.t.x: $\sin\sqrt{\sin\sqrt{x}}$

SOLUTION

Let
$$y = \sin \sqrt{\sin \sqrt{x}}$$

Differentiating w.r.t. x , we get
 $\frac{dy}{dx} = \frac{d}{dx} \left(\sin \sqrt{\sin \sqrt{x}} \right)$
 $= \cos \sqrt{\sin \sqrt{x}} \cdot \frac{d}{dx} \left(\sqrt{\sin \sqrt{x}} \right)$
 $= \cos \sqrt{\sin \sqrt{x}} \times \frac{1}{2\sqrt{\sin \sqrt{x}}} \cdot \frac{d}{dx} (\sin \sqrt{x})$
 $= \frac{\cos \sqrt{\sin \sqrt{x}}}{2\sqrt{\sin \sqrt{x}}} \times \cos \sqrt{x} \cdot \frac{d}{dx} (\sqrt{x})$
 $= \frac{\cos \sqrt{\sin \sqrt{x}} \cdot \cos \sqrt{x}}{2\sqrt{\sin \sqrt{x}}} \times \frac{1}{2\sqrt{x}}$
 $= \frac{\cos \sqrt{\sin \sqrt{x}} \cdot \cos \sqrt{x}}{4\sqrt{x} \cdot \sqrt{\sin \sqrt{x}}}$.
Exercise 1.1 | Q.2.15 | Page 12

Differentiate the following w.r.t.x: $\log \left[\sec \left(e^{x^2} \right) \right]$

SOLUTION





Let
$$y = \log\left[\sec\left(e^{x^2}\right)\right]$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}\log\left[\sec\left(e^{x^2}\right)\right]$
 $= \frac{1}{\sec(e^{x^2})} \cdot \frac{d}{dx}\left[\sec\left(e^{x^2}\right)\right]$
 $= \frac{1}{\sec(e^{x^2})} \cdot \sec\left(e^{x^2}\right) \tan\left(e^{x^2}\right) \cdot \frac{d}{dx}\left(e^{x^2}\right)$
 $= \tan\left(e^{x^2}\right) \cdot e^{x^2} \cdot \frac{d}{dx}(x^2)$
 $= \tan\left(e^{x^2}\right) \cdot e^{x^2} \cdot 2x$
 $= 2x \cdot e^{x^2} \tan\left(e^{x^2}\right)$.

Exercise 1.1 | Q 2.16 | Page 12

Differentiate the following w.r.t.x: loge²(logx) SOLUTION

Let
$$y = \log e^{2}(\log x)$$

$$= \frac{\log(\log x)}{\log e^{2}}$$

$$= \frac{\log(\log x)}{2 \log e}$$

$$= \frac{\log(\log x)}{2} \qquad ...[\because \log e = 1]$$
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{1}{2} \frac{d}{2dx} [\log(\log x)]$$

$$= \frac{1}{2} \times \frac{1}{\log x} \cdot \frac{d}{dx} (\log x)$$



$$= \frac{1}{2\log x} \times \frac{1}{x}$$
$$= \frac{1}{2x\log x}.$$

Exercise 1.1 | Q 2.17 | Page 12

Differentiate the following w.r.t.x: [log {log(logx)}]²

SOLUTION

Let
$$y = [\log \{\log(\log x)\}]^2$$

Differentiating w.r.t. x , we get
 $\frac{dy}{dx} = \frac{d}{dx} [\log\{\log(\log x)\}]^2$
 $= 2. \log\{\log(\log x)\} \times \frac{d}{dx} [\log\{\log(\log x)\}]$
 $= 2. \log\{\log(\log x)\} \times \frac{1}{\log(\log x)} \cdot \frac{d}{dx} [\log(\log x)]$
 $= 2. \log\{\log(\log x)\} \times \frac{1}{\log(\log x)} \cdot \frac{1}{\log x} \times \frac{d}{dx} (\log x)$
 $= 2. \log\{\log(\log x)\} \times \frac{1}{\log(\log x)} \cdot \frac{1}{\log x} \times \frac{1}{x}$
 $= 2. \left[\frac{\log\{\log(\log x)\}}{x \cdot \log(\log x)}\right].$

Exercise 1.1 | Q 2.18 | Page 12

Differentiate the following w.r.t.x: $\sin^2x^2 - \cos^2x^2$





Let
$$y = \sin^2 x^2 - \cos^2 x^2$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} [\sin^2 x^2 - \cos^2 x^2]$
 $= \frac{d}{dx} (\sin x^2)^2 - \frac{d}{dx} (\cos x^2)^2$
 $= 2\sin x^2 \cdot \frac{d}{dx} (\sin x^2) - 2\cos x^2 \cdot \frac{d}{dx} (\cos x^2)$
 $= 2\sin x^2 \cdot \cos x^2 \cdot \frac{d}{dx} (x^2) - 2\cos x^2 \cdot (-\sin x^2) \cdot \frac{d}{dx} (x^2)$
 $= 2\sin x^2 \cdot \cos x^2 x 2x + 2\sin x^2 \cdot \cos x^2 x 2x$
 $= 4x (2\sin x^2 \cdot \cos x^2)$
 $= 4x \sin(2x^2).$

Exercise 1.1 | Q 3.01 | Page 12

Differentiate the following w.r.t.x: $(x^2 + 4x + 1)^3 + (x^3 - 5x - 2)^4$

SOLUTION

Let
$$y = (x^2 + 4x + 1)^3 + (x^3 - 5x - 2)^4$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx} \left[(x^2 + 4x + 1)^3 + (x^3 - 5x - 2)^4 \right]$$

$$= \frac{d}{dx} (x^2 + 4x + 1)^3 + \frac{d}{dx} (x^3 - 5x - 2)^4$$

$$= 3(x^2 + 4x + 1)^2 \cdot \frac{d}{dx} (x^2 + 4x + 1) + 4(x^3 - 5x - 2)^3 \cdot \frac{d}{dx} (x^3 - 5x - 2)$$

$$= 3(x^2 + 4x + 1)^3 \cdot (2x + 4x + 1 + 0) + 4(x^3 - 5x - 2)^3 \cdot (3x^2 - 5x + 1 - 0)$$

$$= 6(x + 2)(x^2 + 4x + 1)^2 + 4(3x^2 - 5)(x^3 - 5x - 2)^3.$$

Exercise 1.1 | Q 3.02 | Page 12 Differentiate the following w.r.t.x: $(1 + 4x)^5 (3 + x - x2)^8$

SOLUTION

Let $y = (1 + 4x)^5 (3 + x - x2)^8$ Differentiating w.r.t. x, we get $\frac{dy}{dx} = \frac{d}{dx} (1 + 4x)^5 (3 + x - x^2)^8$ $= (1 + 4x)^5 \cdot \frac{d}{dx} (3 + x - x^2)^8 + (3 + x - x^2)^8 \cdot \frac{d}{dx} (1 + 4x)^5$ $= (1 + 4x)^5 \times 8(3 + x - x^2)^7 \cdot \frac{d}{dx} (3 + x - x^2) + (3 + x - x^2)^8 \times 5(1 + 4x)^4 \cdot \frac{d}{dx} (1 + 4x)^8$ $= 8(1 + 4x)^5 (3 + x - x^2)^7 \cdot (0 + 1 - 2x) + 5(1 + 4x)^4 (3 + x - x2)^8 \cdot (0 + 4 \times 1)$ $= 8(1 - 2x)(1 + 4x)^5(3 + x - x^2)^7 + 20(1 + 4x)^4(3 + x - x^2)^8$.

Exercise 1.1 | Q 3.03 | Page 12

Differentiate the following w.r.t.x: $\frac{x}{\sqrt{7-3x}}$





Let
$$y = \frac{x}{\sqrt{7-3x}}$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} \left(\frac{x}{\sqrt{7-3x}} \right)$
 $= \frac{\sqrt{7-3x} \cdot \frac{d}{dx}(x) - x \frac{d}{dx}(\sqrt{7-3x})}{(\sqrt{7-3x})^2}$
 $= \frac{\sqrt{7-3x} \times 1 - x \times \frac{1}{2\sqrt{7-3x}} \cdot \frac{d}{dx}(7-3x)}{7-3x}$
 $= \frac{\sqrt{7-3x} - \frac{x}{2\sqrt{7-3x}}(0-3 \times 1)}{7-3x}$
 $= \frac{2(7-3x) + 3x}{2(7-3x)^{\frac{3}{2}}}$
 $= \frac{14-6x+3x}{2(7-3x)^{\frac{3}{2}}}$
 $= \frac{14-3x}{2(7-3x)^{\frac{3}{2}}}$.

Exercise 1.1 | Q 3.04 | Page 12

Differentiate the following w.r.t.x:
$$rac{\left(x^3-5
ight)^5}{\left(x^3+3
ight)^3}$$





Let
$$y = \frac{(x^3 - 5)^5}{(x^3 + 3)^3}$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} \left[\frac{(x^3 - 5)^5}{(x^3 + 3)^3} \right]^2$
 $= \frac{(x^3 + 3)^3 \cdot \frac{d}{dx} (x^3 - 5)^5 - (x^3 - 5)^5 \cdot \frac{d}{dx} (x^3 + 3)^3}{[(x^3 + 3)^3]^2}$
 $= \frac{(x^3 + 3)^3 \times 5(x^3 - 5)^4 \cdot \frac{d}{dx} (x^3 - 5) - (x^3 - 5)^5 \times 3(x^3 + 3)^2 \cdot \frac{d}{dx} (x^3 + 3)}{(x^3 + 3)^6}$
 $= \frac{5(x^3 + 3)^3 (x^3 - 5)^4 \cdot (3x^3 - 0) - 3(x^3 - 5)^5 (x^3 + 3)^2 \cdot (3x^2 + 0)}{(x^3 + 3)^6}$
 $= \frac{3x^2 (x^3 + 3)^2 (x^3 - 5)^4 [5(x^3 + 3) - 3(x^3 - 5)]}{(x^3 + 3)^6}$
 $= \frac{3x^2 (x^3 - 5)^4 (5x^3 + 15 - 3x^3 + 15)}{(x^3 + 3)^4}$
 $= \frac{3x^2 (x^3 - 5)^4 (2x^3 + 30)}{(x^3 + 3)^4}$
 $= \frac{6x^2 (x^2 + 15) (x^3 - 5)^4}{(x^3 + 3)^4}.$

Exercise 1.1 | Q 3.05 | Page 12

Differentiate the following w.r.t.x: $(1 + \sin^2 x)^2 (1 + \cos^2 x)^3$

SOLUTION

Let $y = (1 + \sin^2 x)^2 (1 + \cos^2 x)^3$

Differentiating w.r.t. x, we get

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left[\left(1 + \sin^2 x \right)^2 (1 + \cos^2 x)^3 \right] \\ &= \left(1 + \sin^2 x \right)^2 \cdot \frac{d}{dx} \left(1 + \cos^2 x \right)^3 + \left(1 + \cos^2 x \right)^3 \cdot \frac{d}{dx} \left(1 + \sin^2 x \right)^2 \\ &= \left(1 + \sin^2 x \right)^2 \times 3 \left(1 + \cos^2 x \right)^2 \cdot \frac{d}{dx} \left(1 + \cos^2 x \right) + \left(1 + \cos^2 x \right)^3 \times 2 \left(1 + \sin^2 x \right) \cdot \frac{d}{dx} \left(1 + \sin^2 x \right) \\ &= 3 \left(1 + \sin^2 x \right)^2 \left(1 + \cos^2 x \right)^2 \cdot \left[0 + 2 \cos x \cdot \frac{d}{dx} (\cos x) \right] + 2 \left(1 + \sin^2 x \right) \left(1 + \cos^2 x \right)^3 \cdot \left[0 + 2 \sin x \cdot \frac{d}{dx} (\sin x) \right] \\ &= 3 \left(1 + \sin^2 x \right)^2 \left(1 + \cos^2 x \right)^2 \cdot \left[2 \cos x \left(- \sin x \right) \right] + 2 \left(1 + \sin^2 x \right) \left(1 + \cos^2 x \right)^3 \left[2 \sin x \cdot \cos x \right] \\ &= 3 \left(1 + \sin^2 x \right)^2 \left(1 + \cos^2 x \right)^2 \cdot \left[2 \cos x \left(- \sin x \right) \right] + 2 \left(1 + \sin^2 x \right) \left(1 + \cos^2 x \right)^3 \left[2 \sin x \cdot \cos x \right] \\ &= 3 \left(1 + \sin^2 x \right)^2 \left(1 + \cos^2 x \right)^2 \left(- \sin 2x \right) + 2 \left(1 + \sin^2 x \right) \left(1 + \cos^2 x \right)^3 \left[2 \sin x \cdot \cos x \right] \\ &= 3 \left(1 + \sin^2 x \right)^2 \left(1 + \cos^2 x \right)^2 \left(- 3 \left(1 + \sin^2 x \right) + 2 \left(1 + \cos^2 x \right)^3 \left(\sin 2x \right) \right] \\ &= \sin 2x \left(1 + \sin^2 x \right) \left(1 + \cos^2 x \right)^2 \left[- 3 \left(1 + \sin^2 x \right) + 2 \left(1 + \cos^2 x \right) \right] \\ &= \sin 2x \left(1 + \sin^2 x \right) \left(1 + \cos^2 x \right)^2 \left[- 1 - 3 \sin^2 x + 2 \left(1 - \sin^2 x \right) \right] \\ &= \sin 2x \left(1 + \sin^2 x \right) \left(1 + \cos^2 x \right)^2 \left[- 1 - 3 \sin^2 x + 2 \left(1 - \sin^2 x \right) \right] \\ &= \sin 2x \left(1 + \sin^2 x \right) \left(1 + \cos^2 x \right)^2 \left[- 1 - 3 \sin^2 x + 2 \left(- \sin^2 x \right) \right] \\ &= \sin 2x \left(1 + \sin^2 x \right) \left(1 + \cos^2 x \right)^2 \left[- 1 - 3 \sin^2 x + 2 \left(- 2 \sin^2 x \right) \right] \\ &= \sin 2x \left(1 + \sin^2 x \right) \left(1 + \cos^2 x \right)^2 \left[- 1 - 3 \sin^2 x + 2 \left(- 2 \sin^2 x \right) \right] \\ &= \sin 2x \left(1 + \sin^2 x \right) \left(1 + \cos^2 x \right)^2 \left[- 1 - 3 \sin^2 x + 2 \left(- 2 \sin^2 x \right) \right] \\ &= \sin 2x \left(1 + \sin^2 x \right) \left(1 + \cos^2 x \right)^2 \left[- 1 - 3 \sin^2 x + 2 \left(- 2 \sin^2 x \right) \right] \\ &= \sin 2x \left(1 + \sin^2 x \right) \left(1 + \cos^2 x \right)^2 \left(- 1 - 3 \sin^2 x + 2 \left(- 2 \sin^2 x \right) \right] \\ &= \sin 2x \left(1 + \sin^2 x \right) \left(1 + \cos^2 x \right)^2 \left(1 - 5 \sin^2 x \right) \right)$$

Exercise 1.1 | Q 3.06 | Page 12

Differentiate the following w.r.t.x: $\sqrt{\cos x} + \sqrt{\cos \sqrt{x}}$





Let
$$y = \sqrt{\cos x} + \sqrt{\cos \sqrt{x}}$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} \left[\sqrt{\cos x} + \sqrt{\cos \sqrt{x}} \right]$
 $= \frac{d}{dx} (\cos x)^{\frac{1}{2}} + \frac{d}{dx} (\cos \sqrt{x})^{\frac{1}{2}}$
 $= \frac{1}{2} (\cos x)^{-\frac{1}{2}} \cdot \frac{d}{dx} (\cos x) + \frac{1}{2} (\cos \sqrt{x})^{-\frac{1}{2}} \cdot \frac{d}{dx} (\cos \sqrt{x})$
 $= \frac{1}{2\sqrt{\cos x}} \cdot (-\sin x) + \frac{1}{2\sqrt{\cos \sqrt{x}}} \times (-\sin \sqrt{x}) \cdot \frac{d}{dx} (\sqrt{x})$
 $= \frac{-\sin x}{2\sqrt{\cos x}} - \frac{\sin \sqrt{x}}{2\sqrt{\cos \sqrt{x}}} \times \frac{1}{2\sqrt{x}}$
 $= \frac{-\sin x}{2\sqrt{\cos x}} - \frac{\sin \sqrt{x}}{4\sqrt{x}\sqrt{\cos \sqrt{x}}}.$

Exercise 1.1 | Q 3.07 | Page 12

Differentiate the following w.r.t.x: log (sec 3x+ tan 3x)





Let
$$y = \log(\sec 3x + \tan 3x)$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx}[\log(\sec 3x + \tan 3x)]$$

$$= \frac{1}{\sec 3x + \tan 3x} \cdot \frac{d}{dx}(\sec 3x + \tan 3x)$$

$$= \frac{1}{\sec 3x + \tan 3x} \times \left[\frac{d}{dx}(\sec 3x) + \frac{d}{dx}(\tan 3x)\right]$$

$$= \frac{1}{\sec 3x + \tan 3x} \times \left[\sec 3x \tan 3x \cdot \frac{d}{dx}(3x) + \sec^2 3x \cdot \frac{d}{dx}(3x)\right]$$

$$= \frac{1}{\sec 3x + \tan 3x} \times \left[\sec 3x \tan 3x \cdot 3 + \sec^2 3x \cdot 3\right]$$

$$= \frac{3 \sec 3x(\tan 3x + \sec 3x)}{\sec 3x + \tan 3x}$$

$$= 3 \sec 3x.$$

Exercise 1.1 | Q 3.08 | Page 12

Differentiate the following w.r.t.x: $rac{1+\sin x\,\hat{}}{1-\sin x\,\hat{}}$





$$\begin{aligned} \text{Let } y &= \frac{1 + \sin x^{*}}{1 - \sin x^{*}} \\ &= \frac{1 + \sin(\frac{\pi x}{180})}{1 - \sin(\frac{\pi x}{180})} \quad \dots \left[\because x^{*} = \left(\frac{\pi x}{180}\right)^{*}\right] \\ \text{Differentiating w.r.t. } x, \text{ we get} \\ &\frac{\text{dy}}{\text{dx}} = \frac{\text{d}}{\text{dx}} \left[\frac{1 + \sin(\frac{\pi x}{180})}{1 - \sin(\frac{\pi x}{180})}\right] \\ &= \frac{\left[1 - \sin(\frac{\pi x}{180})\right] \cdot \frac{\text{d}}{\text{dx}} \left[1 + \sin(\frac{\pi x}{180})\right] - \left[1 + \sin(\frac{\pi x}{180})\right] \cdot \frac{\text{d}}{\text{dx}} \left[1 - \sin(\frac{\pi x}{180})\right]}{\left[1 - \sin(\frac{\pi x}{180})\right]^{2}} \\ &= \frac{\left[1 - \sin(\frac{\pi x}{180})\right] \cdot \left[0 + \cos(\frac{\pi x}{180}) \cdot \frac{\text{d}}{\text{dx}} \left(\frac{\pi x}{180}\right) - \left[1 + \sin(\frac{\pi x}{180})\right] \cdot \left[0 - \cos(\frac{\pi x}{180}) \cdot \frac{\text{d}}{\text{dx}} \left(\frac{\pi x}{180}\right)\right]\right)}{\left[1 - \sin(\frac{\pi x}{180})\right]^{2}} \\ &= \frac{\left(1 - \sin x^{*}\right) \left[(\cos x^{*}) \times \frac{\pi}{180} \times 1\right] - (1 + \sin x^{*}) \left[(-\cos x^{*}) \times \frac{\pi}{180} \times 1\right]}{(1 - \sin x^{*})^{2}} \\ &= \frac{\frac{\pi}{180} \cos x^{*} (1 - \sin x^{*} + 1 + \sin x^{*})}{(1 - \sin x^{*})^{2}} \\ &= \frac{\pi \cos x^{*}}{90(1 - \sin x^{*})^{2}}. \end{aligned}$$

Exercise 1.1 | Q 3.09 | Page 12

Differentiate the following w.r.t.x:
$$\cot\left(\frac{\log x}{2}\right) - \log\left(\frac{\cot x}{2}\right)$$





Let
$$y = \cot\left(\frac{\log x}{2}\right) - \log\left(\frac{\cot x}{2}\right)$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} \left[\cot\left(\frac{\log x}{2}\right) - \log\left(\frac{\cot x}{2}\right)\right]$$

$$= \frac{d}{dx} \left[\cot\left(\frac{\log x}{2}\right)\right] - \frac{d}{dx} \left[\log\left(\frac{\cot x}{2}\right)\right]$$

$$= -\csc^{2}\left(\frac{\log x}{2}\right) \cdot \frac{d}{dx}\left(\frac{\log x}{2}\right) - \frac{1}{\left(\frac{\cot x}{2}\right)} \cdot \frac{d}{dx}\left(\frac{\cot x}{2}\right)$$

$$= -\csc^{2}\left(\frac{\log x}{2}\right) \times \frac{1}{2} \times \frac{1}{x} - \frac{2}{\cot x} \times \frac{1}{2} \times (-\csc^{2}x)$$

$$= -\frac{\csc^{2}\left(\frac{\log x}{2}\right)}{2x} + \tan x \cdot \csc^{2}x.$$

Exercise 1.1 | Q 3.1 | Page 12

Differentiate the following w.r.t.x: $rac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$





Let
$$y = \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$$

$$= \frac{e^{2x} - \frac{1}{e^{2x}}}{e^{2x} + \frac{1}{e^{2x}}}$$

$$= \frac{e^{4x} - 1}{e^{4x} + 1}$$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} \left(\frac{e^{4x} - 1}{e^{4x} + 1} \right)$

$$= \frac{(e^{4x} + 1) \cdot \frac{d}{dx} (e^{4x} - 1) - (e^{4x} - 1) \cdot \frac{d}{dx} (e^{4x} + 1)}{(e^{4x} + 1)^2}$$

$$= \frac{(e^{4x} + 1) \left[e^{4x} \cdot \frac{d}{dx} (4x) - 0 \right] - (e^{4x} - 1) \left[e^{4x} \cdot \frac{d}{dx} (4x) + 0 \right]}{(e^{4x} + 1)^2}$$

$$= \frac{(e^{4x} + 1) \cdot e^{4x} \times 4 - (e^{4x} - 1) \cdot e^{4x} \times 4}{(e^{4x} + 1)^2}$$

$$= \frac{4e^{4x} (e^{4x} + 1 - e^{4x} + 1)}{(e^{4x} + 1)^2}$$

$$= \frac{8e^{4x}}{(e^{4x} + 1)^2}.$$

Exercise 1.1 | Q 3.11 | Page 12

Differentiate the following w.r.t.x: $rac{e^{\sqrt{x}}+1}{e^{\sqrt{x}}-1}$



Let y = $\frac{e^{\sqrt{x}} + 1}{e^{\sqrt{x}} - 1}$ Differentiating w.r.t. x, we get $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{d}}{\mathrm{dx}} \left(\frac{e^{\sqrt{x}} + 1}{e^{\sqrt{x}} - 1} \right)$ $=\frac{\left(e^{\sqrt{x}}-1\right)\frac{\mathrm{d}}{\mathrm{dx}}\left(e^{\sqrt{x}}+1\right)-\left(e^{\sqrt{x}}+1\right)\frac{\mathrm{d}}{\mathrm{dx}}\left(e^{\sqrt{x}}-1\right)}{\left(e^{\sqrt{x}}-1\right)^{2}}$ $=\frac{\left(e^{\sqrt{x}}-1\right)\left[e^{\sqrt{x}}\cdot\frac{\mathrm{d}}{\mathrm{dx}}\left(\sqrt{x}\right)+0\right]-\left(e^{\sqrt{x}}+1\right)\left[e^{\sqrt{x}}\cdot\frac{\mathrm{d}}{\mathrm{dx}}\left(\sqrt{x}\right)-0\right]}{\left(e^{\sqrt{x}}-1\right)^{2}}$ $=\frac{\left(e^{\sqrt{x}}-1\right)\left[e^{\sqrt{x}}\times\frac{1}{2\sqrt{x}}\right]-\left(e^{\sqrt{x}}+1\right)\left[e^{\sqrt{x}}\times\frac{1}{2\sqrt{x}}\right]}{\left(e^{\sqrt{x}}-1\right)^{2}}$ $=\frac{\frac{e^{\sqrt{x}}}{2\sqrt{x}}\left(e^{\sqrt{x}}-1-e^{\sqrt{x}}-1\right)}{\left(e^{\sqrt{x}}-1\right)^2}$ $=\frac{-e^{\sqrt{x}}}{\sqrt{x}(e^{\sqrt{x}}-1)^2}.$

Exercise 1.1 | Q 3.12 | Page 12

Differentiate the following w.r.t.x: $\log[\tan^3 x.\sin^4 x.(x^2 + 7)^7]$





Let
$$y = \log[\tan^3 x . \sin^4 x. (x^2 + 7)^7]$$

= $\log \tan^3 x + \log \sin^4 x + \log(x^2 + 7)^7$
= $3\log \tan x + 4\log \sin x + 7\log(x^2 + 7)$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} [3\log \tan x + 4\log \sin x + 7\log(x^2 + 7)]$
= $3\frac{d}{dx}(\log \tan x) + 4\frac{d}{dx}(\log \sin x) + 7\frac{d}{dx}[\log(x^2 + 7)]$
= $3 \times \frac{1}{\tan x} \cdot \frac{d}{dx}(\tan x) + 4 \times \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x) + 7 \times \frac{1}{x^2 + 7} \cdot \frac{d}{dx}(x^2 + 7)$
= $3 \times \frac{1}{\tan x} \cdot \sec^2 x + 4 \times \frac{1}{\sin x} \cdot \cos x + 7 \times \frac{1}{x^2 + 7} \cdot (2x + 0)$
= $3 \times \frac{\cos x}{\sin x} \times \frac{1}{\cos^2 x} + 4 \cot x + \frac{14x}{x^2 + 7}$
= $\frac{6}{\sin 2x} + 4 \cot x + \frac{14x}{x^2 + 7}$
= $\frac{6}{\sin 2x} + 4 \cot x + \frac{14x}{x^2 + 7}$

Exercise 1.1 | Q 3.13 | Page 12

Differentiate the following w.r.t.x: $\log\left(\sqrt{rac{1-\cos 3x}{1+\cos 3x}}
ight)$





Let
$$y = \log\left(\sqrt{\frac{1-\cos 3x}{1+\cos 3x}}\right)$$

$$= \log\left(\sqrt{\frac{2\sin^3\left(\frac{3x}{2}\right)}{2\cos^2\left(\frac{3x}{2}\right)}}\right)$$

$$= \log \tan\left(\frac{3x}{2}\right)$$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log \tan\left(\frac{3x}{2}\right)\right]$$

$$= \frac{1}{\tan\left(\frac{3x}{2}\right)} \times \frac{d}{dx}\left[\tan\left(\frac{3x}{2}\right)\right]$$

$$= \frac{1}{\tan\left(\frac{3x}{2}\right)} \times \sec^2\left(\frac{3x}{2}\right) \cdot \frac{d}{dx}\left(\frac{3x}{2}\right)$$

$$= \frac{\cos\left(\frac{3x}{2}\right)}{\sin\left(\frac{3x}{2}\right)} \times \frac{1}{\cos^2\left(\frac{3x}{2}\right)} \times \frac{3}{2} \times 1$$

$$= 3 \times \frac{1}{2\sin\left(\frac{3x}{2}\right)\cos\left(\frac{3x}{2}\right)}$$

$$= 3 \times \frac{1}{\sin 3x}$$

$$= 3 \operatorname{cosec3x}.$$
Exercise 1.1 [Q3.14] Page 12
Differentiate the following w.r.t.x: $\left(\sqrt{\frac{1+\cos\left(\frac{5x}{2}\right)}{1-\cos\left(\frac{5x}{2}\right)}}\right)$

SOLUTION

$$\begin{array}{l} \text{Using}\\ \log\left(\frac{a}{b}\right) = \log a - \log b\\ \log a^b = b \log a\\ y = \log\left(\sqrt{1 + \cos\left(\frac{5x}{2}\right)} - \log\left(\sqrt{1 - \cos\left(\frac{5x}{2}\right)}\right)\\ y = \log\left(1 + \cos\left(\frac{x}{2}\right)^{\frac{1}{2}} - \log\left(1 - \cos\left(\frac{5x}{2}\right)\right)^{\frac{1}{2}}\\ y = \frac{1}{2}\log\left[1 + \cos\left(\frac{5x}{2}\right)\right] - \frac{1}{2}\log\left[\left(1 - \cos\left(\frac{5x}{2}\right)\right)\\ \text{Differentiating w.r.t. x}\\ \frac{dy}{dx} = \frac{1}{2}\frac{1}{1 + \cos\left(\frac{5x}{2}\right)}\frac{d}{dx}\left(1 + \cos\left(\frac{5x}{2}\right) - \frac{1}{2} \times \frac{1}{1 - \cos\left(\frac{5x}{2}\right)}\frac{d}{dx}\left(1 - \cos\left(\frac{5x}{2}\right)\right)\\ = \frac{1}{2(1 + \cos\left(\frac{5x}{2}\right))}\left(-\sin\left(\frac{5x}{2}\right) \cdot \frac{5}{2} - \frac{1}{2(1 - \cos\left(\frac{5x}{2}\right))}\left(\sin\left(\frac{5x}{2}\right) \cdot \frac{5}{2}\right)\\ = \frac{-5\sin\left(\frac{5x}{2}\right)}{4(1 + \cos\left(\frac{5x}{2}\right)} - \frac{5\sin\left(\frac{5x}{2}\right)}{4(1 - \cos\left(\frac{5x}{2}\right))} \\ = \frac{-5}{4}\sin\left(\frac{5x}{2}\right)\left[\frac{1}{1 + \cos\left(\frac{5x}{2}\right)} + \frac{1}{1 - \cos\left(\frac{5x}{2}\right)}\right]\\ = \frac{-5}{4}\sin\left(\frac{5x}{2}\right) \times \frac{2}{\sin^2\left(\frac{5x}{2}\right)} \dots \left[\because 1 - \cos^2x = \sin^2x\right]\\ = \frac{-5}{4}\frac{1}{\sin\left(\frac{5x}{2}\right)}\\ = \frac{-5}{2}\csc\left(\frac{5x}{2}\right). \end{array}$$

Exercise 1.1 | Q 3.15 | Page 12

Differentiate the following w.r.t.x: $\log \left(\sqrt{rac{1-\sin x}{1+\sin x}}
ight)$

SOLUTION

Let
$$y = \log\left(\sqrt{\frac{1-\sin x}{1+\sin x}}\right)$$

$$= \log\left(\sqrt{\frac{1-\sin x}{1+\sin x}} \times \frac{1-\sin x}{1-\sin x}\right)$$

$$= \log\left(\sqrt{\frac{(1-\sin x)^2}{1-\sin^2 x}}\right)$$

$$= \log\left(\sqrt{\frac{(1-\sin x)^2}{\cos^2 x}}\right)$$

$$= \log\left(\frac{1-\sin x}{\cos x}\right)$$

$$= \log\left(\frac{1-\sin x}{\cos x}\right)$$

$$= \log\left(\frac{1-\sin x}{\cos x}\right)$$

$$= \log(\sec x - \tan x)$$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx} [\log(\sec x - \tan x)]$$





$$= \frac{1}{\sec x - \tan x} \cdot \frac{d}{dx} (\sec x - \tan x)$$
$$= \frac{1}{\sec x - \tan x} \times (\sec x \tan x - \sec^2 x)$$
$$= \frac{-\sec x (\sec x - \tan x)}{\sec x - \tan x}$$
$$= -\sec x.$$

Exercise 1.1 | Q 3.16 | Page 12

Differentiate the following w.r.t.x: $\log \left[4^{2x} \left(\frac{x^2 + 5}{\sqrt{2x^3 - 4}} \right)^{\frac{3}{2}} \right]$

SOLUTION

Let
$$y = \log \left[4^{2x} \left(\frac{x^2 + 5}{\sqrt{2x^3 - 4}} \right)^{\frac{3}{2}} \right]$$

$$= \log 4^{2x} + \log \left(\frac{x^2 + 5}{\sqrt{2x^3 - 4}} \right)^{\frac{3}{2}}$$

$$= 2x \log 4 + \frac{3}{2} \log \left(\frac{x^2 + 5}{\sqrt{2x^3 - 4}} \right)$$

$$= 2x \log 4 + \frac{3}{2} \left[\log(x^2 + 5) - \log(2x^3 - 4)^{\frac{1}{2}} \right]$$

$$= 2x \log 4 + \frac{3}{2} \left[\log(x^2 + 5) - \log(2x^3 - 4) \right]$$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx} \left[2x \log 4 + \frac{3}{2} \log(x^2 + 5) - \frac{3}{4} \log(2x^3 - 4) \right]$$

$$= (2 \log 4) \frac{d}{dx} (x) + \frac{3}{2} \frac{d}{dx} \left[\log(x^2 + 5) \right] - \frac{3}{4} \frac{d}{dx} \left[\log(2x^3 - 4) \right]$$





$$= (2 \log 4) \times 1 + \frac{3}{2} \times \frac{1}{x^2 + 5} \cdot \frac{d}{dx} (x^2 + 5) - \frac{3}{4} \times \frac{1}{2x^3 - 4} \cdot \frac{d}{dx} (2x^3 - 4)$$

$$= 2 \log 4 + \frac{3}{2(x^2 + 5)} \times (2x + 0) - \frac{3}{4(2x^3 - 4)} \times (2 \times 3x^2 - 0)$$

$$= 2 \log 4 + \frac{3x}{x^2 + 5} - \frac{9x^2}{2(2x^3 - 4)}.$$

Exercise 1.1 | Q 3.17 | Page 12

Differentiate the following w.r.t.x:
$$\log \left[rac{ex^2(5-4x)^{rac{3}{2}}}{\sqrt[3]{7-6x}}
ight]$$

SOLUTION

Let y =
$$\log \left[\frac{e^{x^2} (5 - 4x)^{\frac{3}{2}}}{\sqrt[3]{7 - 6x}} \right]$$

Using

$$log(A.B) = logA + logB$$

$$y = log e^{x^{2}} + log\left(\frac{(5-4x)^{\frac{3}{2}}}{\sqrt[3]{7-6x}}\right)$$

$$= log e^{x^{2}} + log(5-4x)^{\frac{3}{2}} - log\left(\sqrt[3]{7-6x}\right)$$

$$= x^{2} log e + \frac{3}{2} log(5-4x) - log(7-6x)^{\frac{1}{3}}$$

$$= x^{2} + \frac{3}{2} log(5-4x) - \frac{1}{3} log(7-6x)$$
Now,
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{d}{dx}x^{2} + \frac{3}{2}\frac{d}{dx} log(5-4x) - \frac{1}{3}\frac{d}{dx} log(7-6x)$$

$$= 2x + \frac{3}{2}\frac{1}{5-4x}(-4) - \frac{1}{3}\frac{1}{(7-6x)}x(-6)$$





$$=2x-rac{6}{(5-4x)}+rac{2}{(7-6x)}$$
 $2x-rac{6}{5-4x}+rac{2}{7-6x}.$

Exercise 1.1 | Q 3.18 | Page 12

Differentiate the following w.r.t.x:

$$\log \! \left[rac{a^{\cos x}}{\left(x^2-3
ight)^3 \log x}
ight]$$

SOLUTION

Let $y = \log \left[\frac{a^{\cos x}}{(x^2 - 3)^3 \log x} \right]$ = $\log a^{\cos x} - \log(x^2 - 3)^3 - \log(\log x)$ = $(\cos x)(\log a) - 3\log(x^2 - 3) - \log(\log x)$ Differentiating w.r.t. x, we get $\frac{dy}{dx} = \frac{d}{dx} \left[(\cos x)(\log a) - 3\log(x^2 - 3) - \log(\log x) \right]$ = $(\log a) \cdot \frac{d}{dx} (\cos x) - 3\frac{d}{dx} \left[\log(x^2 - 3) \right] - \frac{d}{dx} [\log(\log x)]$ = $(\log a)(-\sin x) - 3 \times \frac{1}{x^2 - 3} \cdot \frac{d}{dx} (x^2 - 3) - \frac{1}{\log x} \cdot \frac{d}{dx} (\log x)$ = $-(\sin x)(\log a) - \frac{3}{x^2 - 3} \times (2x - 0) - \frac{1}{\log x} \times \frac{1}{x}$ = $-(\sin x)(\log a) - \frac{6x}{x^2 - 3} - \frac{1}{x\log x}$. Exercise 1.1 [Q.3.19] Page 12

Differentiate the following w.r.t.x: y = $(25)^{\log_5(\sec x)} - (16)^{\log_4(\tan x)}$



$$y = (25)^{\log_5(\sec x)} - (16)^{\log_4(\tan x)}$$

= $5^{2\log_5(\sec x)} - 4^{2\log_4(\tan x)}$
= $5^{2\log_5(\sec^2 x)} - 4^{2\log_4(\tan^2 x)}$
= $\sec^2 x - \tan^2 x$...[:: $a^{\log_a x} = x$]
 $\therefore y = 1$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}(1) = 0.$

Exercise 1.1 | Q 3.2 | Page 12

Differentiate the following w.r.t.x: $rac{\left(x^2+2
ight)^4}{\sqrt{x^2+5}}$





$$\begin{aligned} &\text{Let } \mathbf{y} = \frac{\left(x^2 + 2\right)^4}{\sqrt{x^2 + 5}} \\ &\text{Differentiating w.r.t. x, we get} \\ &\frac{d\mathbf{y}}{d\mathbf{x}} = \frac{d}{d\mathbf{x}} \left[\frac{\left(x^2 + 2\right)^4}{\sqrt{x^2 + 5}} \right] \\ &= \frac{\sqrt{x^2 + 5} \cdot \frac{d}{d\mathbf{x}} \left(x^2 + 2\right)^4 - \left(x^2 + 2\right)^4 \cdot \frac{d}{d\mathbf{x}} \left(\sqrt{x^2 + 5}\right)}{\left(\sqrt{x^2 + 5}\right)^2} \\ &= \frac{\sqrt{x^2 + 5} \times 4 \left(x^2 + 2\right)^3 \cdot \frac{d}{d\mathbf{x}} \left(x^2 + 2\right) - \left(x^2 + 2\right)^4 \times \frac{1}{2\left(\sqrt{x^2 + 5}\right)} \cdot \frac{d}{d\mathbf{x}} \left(x^2 + 5\right)}{x^2 + 5} \\ &= \frac{\sqrt{x^2 + 5} \times 4 \left(x^2 + 2\right)^3 \cdot \left(2x + 0\right) - \frac{\left(x^2 + 2\right)^4}{2\sqrt{x^2 + 5}} \times \left(2x + 0\right)}{x^2 + 5} \\ &= \frac{8x \left(x^2 + 5\right) \left(x^2 + 2\right)^3 - x \left(x^2 + 2\right)^4}{\left(x^2 + 5\right)^{\frac{3}{2}}} \\ &= \frac{x \left(x^2 + 2\right)^3 \left[8 \left(x^2 + 5\right) - \left(x^2 + 2\right)\right]}{\left(x^2 + 5\right)^{\frac{3}{2}}} \\ &= \frac{x \left(x^2 + 2\right)^3 \left(8x^2 + 40 - x^2 - 2\right)}{\left(x^2 + 5\right)^{\frac{3}{2}}} \\ &= \frac{x \left(x^2 + 2\right)^3 \left(7x^2 + 38\right)}{\left(x^2 + 5\right)^{\frac{3}{2}}}. \end{aligned}$$

Exercise 1.1 | Q 4.1 | Page 12

A table of values of f, g, f' and g' is given :

x f(x) g(x) f'(x) g'(x)	
-------------------------	--

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>>>

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2	1	6	-3	4
4	3	4	5	-6
6	5	2	-4	7

If r(x) = f[g(x)] find r'(2).

SOLUTION

$$\begin{aligned} r(x) &= f[g(x)] \\ \therefore r'(x) &= \frac{d}{dx} f[g(x)] \\ &= f'[g(x)] \cdot \frac{d}{dx} [g(x)] \\ &= f'[g(x)] \cdot [g'(x)] \\ \therefore r'(2) &= f'[g(2)] \cdot g'(2) \\ &= f'(6) \cdot g'(2) \qquad \dots [\because g(x) = 6, \text{ when } x = 2] \\ &= -4 \times 4 \qquad \dots [\text{From the table}] \\ &= -16. \end{aligned}$$

Exercise 1.1 | Q 4.2 | Page 12

A table of values of f, g, f' and g' is given :

x	f(x)	g(x)	f'(x)	g'(x)
2	1	6	-3	4
4	3	4	5	-6
6	5	2	-4	7

If R(x) = g[3 + f(x)] find R'(4).



$$R(x) = g[3 + f(x)]$$

$$\therefore R'(x) = \frac{d}{dx} \{g[3 + f(x)]\}$$

$$= g'[3 + f(x)] \cdot \frac{d}{dx}[3 + f(x)]$$

$$= g'[3 + f(x)] \cdot [0 + f'(x)]$$

$$= g'[3 + f(x)] \cdot f'(x)$$

$$\therefore R'(4) = g'[3 + f(4)] \cdot f'(4)$$

$$= g'[3 + 3] \cdot f'(4) \qquad \dots [\because f(x) = 3, \text{ when } x = 4]$$

$$= g'(6) \cdot f'(4)$$

$$= 7 \times 5 \qquad \dots [\text{Frrom the table}]$$

$$= 35.$$

Exercise 1.1 | Q 4.3 | Page 12

A table of values of f, g, f' and g' is given :

x	f(x)	g(x)	f'(x)	g'(x)
2	1	6	-3	4
4	3	4	5	-6
6	5	2	-4	7

If s(x) = f[9 - f(x)] find s'(4).





$$\begin{aligned} s(x) &= f[9 - f(x)] \\ \therefore s'(x) &= \frac{d}{dx} \{ f[9 - f(x)] \} \\ &= f'[9 - f(x)] \cdot \frac{d}{dx} [9 - f(x)] \\ &= f'[9 - f(x)] \cdot [0 - f'(x)] \\ &= - f'[9 - f(x)] \cdot f'(x) \\ \therefore s'(4) &= - f'[9 - f(4)] \cdot f'(4) \\ &= - f'[9 - 3] \cdot f'(4) \qquad \dots [\because f(x) = 3, \text{ when } x = 4] \\ &= - f'(6) \cdot f'(4) \\ &= - (-4)(5) \qquad \dots [\text{From the table}] \\ &= 20. \end{aligned}$$

Exercise 1.1 | Q 4.4 | Page 12

A table of values of f, g, f' and g' is given :

x	f(x)	g(x)	f'(x)	g'(x)
2	1	6	-3	4
4	3	4	5	-6
6	5	2	-4	7

If S(x) = g[g(x)] find S'(6).







$$S(x) = g[g(x)]$$

$$\therefore S'(x) = \frac{d}{dx}g[g(x)]$$

$$= g'[g(x)] \cdot \frac{d}{dx}[g(x)]$$

$$= g'[g(x)] \cdot g'(x)$$

$$\therefore S'(6) = g'[g(6) \cdot g'(6)]$$

$$= g'(2) \cdot g'(6) \qquad \dots[\because g(x) = 2, \text{ when } x = 6]$$

$$= 4 \times 7 \qquad \dots[\text{From the table}]$$

$$= 28.$$

Exercise 1.1 | Q 5 | Page 12

Assume that
$$f'(3) = -1, g'(2) = 5, g(2) = 3$$
 and $y = f[g(x)], then \left[\frac{dy}{dx}\right]_{x=2} = ?$

$$y = f[g(x)]$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} \{f[g(x)]\}$$

$$= f'[g(x)] \cdot \frac{d}{dx}[g(x)]$$

$$= f'[g(x) \cdot g'(x)$$

$$\therefore \left[\frac{dy}{dx}\right]_{x=2}$$

$$= f'[g(2)] \cdot g'(2)$$

$$= f'(3) \cdot g'(2) \qquad \dots [\because g(2) = 3]$$

$$= -1 \times 5 \qquad \dots (Given)$$

$$= -5.$$

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Exercise 1.1 | Q 6 | Page 12

If h(x) =
$$\sqrt{4f(x) + 3g(x)}, f(1) = 4, g(1) = 3, f'(1) = 3, g'(1) = 4, \text{find h}'(1)$$
.

SOLUTION

Given :
$$f(1) = 4$$
, $g(1) = 3$, $f'(1) = 3$, $g'(1) = 4$...(1)
Now, $h(x) = \sqrt{4f(x) + 3g(x)}$
 $\therefore h'(x) = \frac{d}{dx} \left[\sqrt{4f(x) + 3g(x)} \right]$
 $= \frac{1}{2\sqrt{4f(x) + 3g(x)}} \cdot \frac{d}{dx} [4f(x) + 3g(x)]$
 $= \frac{1}{2\sqrt{4f(x) + 3g(x)}} \cdot [4f'(x) + 3g'(x)]$
 $\therefore h'(1) = \frac{1}{2\sqrt{4f(1) + 3g(1)}} \cdot [4f'(1) + 3g'(1)]$
 $= \frac{1}{2\sqrt{4 \times 4 + 3 \times 3}} \times [4 \times 3 + 3 \times 4]$...[By (1)]
 $= \frac{1}{2\sqrt{25}} \times 24$
 $= \frac{1}{2 \times 5} \times 24$
 $= \frac{12}{5}$.

Exercise 1.1 | Q 7 | Page 12

Find the x co-ordinates of all the points on the curve $y = \sin 2x - 2 \sin x$, $0 \le x < 2\pi$, where dy/dx = 0.





$$y = \sin 2x - 2 \sin x, 0 \le x < 2\pi$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} (\sin 2x - 2 \sin x)$$

$$= \frac{d}{dx} (\sin 2x) - 2 \frac{d}{dx} (\sin x)$$

$$= \cos 2x. \frac{d}{dx} (2x) - 2 \cos x$$

$$= \cos^2 x x 2 - 2 \cos x$$

$$= 2 (2 \cos^2 x - 1) - 2 \cos x$$

$$= 4 \cos^2 x - 2 - 2 \cos x$$

$$= 4 \cos^2 x - 2 - 2 \cos x - 2$$

If $\frac{dy}{dx} = 0$, then $4 \cos^2 x - 2 \cos x - 2 = 0$

$$\therefore 4 \cos^2 x - 4 \cos x 2 \cos x - 2 = 0$$

$$\therefore 4 \cos^2 x - 4 \cos x 2 \cos x - 2 = 0$$

$$\therefore 4 \cos x (\cos x - 1) + 2 (\cos x - 1) = 0$$

$$\therefore (\cos x - 1)(4 \cos x + 2) =$$

$$\therefore \cos x - 1 = 0 \text{ or } 4 \cos x + 2 = 0$$

$$\therefore \cos x = 1 \text{ or } \cos x = -\frac{1}{2}$$

$$\therefore \cos x = \cos 0$$

or

$$\cos x = -\cos \frac{\pi}{3}$$

$$= \cos(\pi - \frac{\pi}{3})$$

$$= \frac{\cos 2\pi}{3}$$

or

$$\cos x = -\cos \frac{\pi}{3}$$



$$= \cos\left(\pi - \frac{\pi}{3}\right)$$

$$= \frac{\cos 4\pi}{3} \qquad \dots [\because 0 \le x < 2\pi]$$

$$\therefore x = 0 \text{ or } x = \frac{2\pi}{3} \text{ or } x = \frac{4\pi}{3}.$$

$$\therefore x = 0 \text{ or } \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}.$$

Exercise 1.1 | Q 8 | Page 13

Select the appropriate hint from the hint basket and fill up the blank spaces in the following paragraph. [Activity]:

"Let f (x) = x^2 + 5 and g (x) = e^x + 3 then f[g(x)] = and g[f(x)] =.....

Now $f'(x) = \dots$ and $g'(x) = \dots$ The derivative of f[g(x)] w. r. t. x in terms of f and g is

Therefore
$$\frac{d}{dx}[f[g(x)]] = \dots$$
 and
 $\left[\frac{d}{dx}[f[g(x)]]\right]_{x=0} = \dots$
The derivative of $g[f(x)]$ w. r. t. x in terms of f and g is
Therefore $\frac{d}{dx}[g[f(x)]] = \dots$ and
 $\left[\frac{d}{dx}[g[f(x)]]\right]_{x=-1} = \dots$
Hint basket : $\left\{f'[g(x)] \cdot g'(x), 2e^{2x} + 6e^x, 8, g'[f(x)] \cdot f'(x), 2xe^{x^2+5}, -2e^6, e^{2x} + 6e^x + 14, e^{x^2+5} + 3, 2x, e^x\right\}$





 $\begin{aligned} f[g(x)] &= e^{2x} + 6e^x + 14 \\ g[f(x)] &= e^{x^2 + 5} + 3 \\ f'(x) &= 2x, \quad g'f(x) = e^x \end{aligned}$

The derivative of f[g(x)] w. r. t. x in terms of f and g is f'[g(x)]. g'(x).

$$\therefore \frac{\mathrm{d}}{\mathrm{d}x} \{f[\mathrm{g}(x)]\} = 2e^{2x} + 6e^x \ \text{and} \ \frac{\mathrm{d}}{\mathrm{d}x} \{f[\mathrm{g}(x)]\}_{x=0} = 8$$

The derivative of g[f(x)] w. r. t. x in terms of f and g is g'[f(x)]. f'(x).

$$\therefore \frac{\mathrm{d}}{\mathrm{d}x} \{ \mathrm{g}[f(x)] \} = 2e^{x^2+5} \text{ and}$$
$$\frac{\mathrm{d}}{\mathrm{d}x} \{ \mathrm{g}[f(x)] \}_{x=-1} = -2\mathrm{e}^{6}.$$

EXERCISE 1.2 [PAGES 29 - 30]

Exercise 1.2 | Q 1.1 | Page 29

Find the derivative of the function y = f(x) using the derivative of the inverse function $x = f^{-1}(y)$ in the following: $y = \sqrt{x}$





 $y = \sqrt{y}$

We have to find the inverse function of y = f(x), i.e. x in terms of y.

From (1),

$$y^{2} = x$$

$$\therefore x = y^{2}$$

$$\therefore x = f^{-1}(y) = y^{2}$$

$$\therefore \frac{dx}{dy} = \frac{d}{dy}(y^{2}) = 2y$$

$$= 2\sqrt{x} \qquad \dots[By (1)]$$

$$\therefore \frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$$

$$= \frac{1}{2\sqrt{x}}.$$

Exercise 1.2 | Q 1.2 | Page 29

Find the derivative of the function y = f(x) using the derivative of the inverse function x =

f⁻¹(y) in the following: y =
$$\sqrt{2-\sqrt{x}}$$





 $y = \sqrt{2 - \sqrt{x}}$...(1) We have to find the inverse function of y = f(x), i.e x in terms of y. From (1), $y^2 = 2 - \sqrt{x}$ $\therefore \sqrt{x} = 2 - u^2$ $\therefore x = (2 - v^2)^2$ $\therefore x = f^{-1}(y) = (2 - y^2)^2$ $\therefore \frac{\mathrm{d}x}{\mathrm{d}y} = \frac{\mathrm{d}}{\mathrm{d}y} \left(2 - y^2\right)^2$ $= 2(2-y^2) \cdot \frac{\mathrm{d}}{\mathrm{d} \mathrm{v}} (2-y^2)$ $= 2(2 - y^2).(0 - 2y)$ $= -4v(2 - v^2)$ $= -4\sqrt{2-\sqrt{x}}(2-2+\sqrt{x})$...[By (1)] $= -4\sqrt{x}\sqrt{2-\sqrt{x}}$ $\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{\left(\frac{\mathrm{dx}}{\mathrm{dy}}\right)} \\ = \frac{1}{4\sqrt{x}\sqrt{2-\sqrt{x}}}.$

Exercise 1.2 | Q 1.3 | Page 29





Find the derivative of the function y = f(x) using the derivative of the inverse function $x = f^{1}(y)$ in the following: $y = \sqrt[3]{x-2}$

SOLUTION

$$y = \sqrt[3]{x-2}$$
 ...(1)

We have to find the inverse function of y = f(x), i.e x in terms of y.

$$y^{3} = x - 2$$

$$\therefore x = y^{3} + 2$$

$$\therefore x = f^{-1}(y) = y^{3} + 2$$

$$\therefore \frac{dx}{dy} = \frac{d}{dy}(y^{3} + 2)$$

$$= 3y^{2} + 0 = 3y^{2}$$

$$= 3\left(\sqrt[3]{(x-2)}\right)^{2} \qquad \dots [By (1)]$$

$$= 3(x - 2)^{\frac{2}{3}}$$

$$= 3.\left(\sqrt[3]{(x-2)^{2}}\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$$

$$= \frac{1}{3\sqrt[3]{(x-2)^{2}}}, x > 2.$$

Exercise 1.2 | Q 1.4 | Page 29

Find the derivative of the function y = f(x) using the derivative of the inverse function $x = f^{-1}(y)$ in the following: y = log(2x - 1)





 $y = \log(2x - 1)$...(1) We have to find the inverse function of y = f(x), i.e x in terms of y. From (1), $2x - 1 = e^{y}$ $\therefore 2x = e^y + 1$ $\therefore x = f^{-1}(v)$ $=\frac{1}{2}(e^{y}+1)$ $\therefore \frac{\mathrm{d} \mathrm{x}}{\mathrm{d} \mathrm{v}} = \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d} \mathrm{v}} (e^{y} + 1)$ $=\frac{1}{2}(e^{y}+0)$ $=\frac{1}{2}e^{y}$ $=\frac{1}{2}e^{\log(2x-1)}$...[By (1)] $=\frac{1}{2}(2x-1)$...[$\because e^{\log x} = x$] $\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{\left(\frac{\mathrm{dx}}{\mathrm{dy}}\right)}$ $=\frac{2}{2r-1}.$

Exercise 1.2 | Q 1.5 | Page 29

Find the derivative of the function y = f(x) using the derivative of the inverse function $x = f^{-1}(y)$ in the following: y = 2x + 3





y = 2x + 3

We have to find the inverse function of y = f(x), i.e x in terms of y.

...(1)

From (1), 2x = y - 3 $\therefore x = \frac{y - 3}{2}$ $\therefore x = f^{-1}(y)$ $= \frac{y - 3}{2}$ $\therefore \frac{dx}{dy} = \frac{1}{2} \frac{d}{dy} (y - 3)$ $= \frac{1}{2} (1 - 0)$ $= \frac{1}{2}$ $\therefore \frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$ $= \frac{1}{\left(\frac{1}{2}\right)}$ = 2.

Exercise 1.2 | Q 1.6 | Page 29

Find the derivative of the function y = f(x) using the derivative of the inverse function $x = f^{-1}(y)$ in the following: $y = e^x - 3$





 $y = e^{x} - 3$...(1) We have to find the inverse function of y = f(x), i.e x in terms of y. From (1), $e^{x} = y + 3$ $\therefore x = \log(y + 3)$ $\therefore x = f^{-1}(y) = \log(y + 3)$ $\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}y} [\log(y+3)]$ $=\frac{1}{y+3}\cdot\frac{\mathrm{d}}{\mathrm{d}y}(y+3)$ $=\frac{1}{u+3}.(1+0)$ $=\frac{1}{y+3}$ $=\frac{1}{e^x-3+3}$...[By (1)] $= \frac{1}{\frac{e^x}{dx}}$ $\therefore \frac{\frac{dy}{dx}}{\frac{dy}{dx}} = \frac{1}{\left(\frac{dx}{dy}\right)}$ $=\frac{1}{\left(\frac{1}{x}\right)}$ = e^x.

Exercise 1.2 | Q 1.7 | Page 29

Find the derivative of the function y = f(x) using the derivative of the inverse function $x = f^{-1}(y)$ in the following: $y = e^{2x-3}$





 $y = e^{2x-3}$...(1) We have to find the inverse function of y = f(x), i.e x in terms of y. From (1), $2x - 3 = \log y$ $\therefore 2x = \log y + 3$ $\therefore x = f^{-1}(y)$ $=\frac{1}{2}(\log y+3)$ $\therefore \frac{\mathrm{dx}}{\mathrm{dy}} = \frac{1}{2} \frac{\mathrm{d}}{\mathrm{dy}} (\log y + 3)$ $=\frac{1}{2}\left(\frac{1}{u}+0\right)$ $=\frac{1}{2u}$ $=\frac{1}{2e^{2x-3}}$...[By (1)] $\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{\left(\frac{\mathrm{dx}}{\mathrm{dy}}\right)}$ $=\frac{1}{\left(\frac{1}{2e^{2x-3}}\right)}$ $= 2e^{2x-3}$.

Exercise 1.2 | Q 1.8 | Page 29

Find the derivative of the function y = f(x) using the derivative of the inverse function $x = \int_{1}^{1} f^{-1}(y)$ in the following: $y = \int_{1}^{1} g(x) dx$





$$y = \log_2\left(\frac{x}{2}\right) \qquad \dots(1)$$

We have to find the inverse function of $y = f(x)$, i.e x in terms of y.
From (1),
$$\frac{x}{2} = 2^{y}$$
$$\therefore x = 2.2^{y} = 2^{y+1}$$
$$\therefore \frac{dx}{dy} = \frac{d}{dy}(2^{y+1})$$
$$= 2^{y+1} \cdot \log 2 \cdot \frac{d}{dy}(y+1)$$
$$= 2^{y+1} \cdot \log 2 \cdot (1+0)$$
$$= 2^{y+1} \cdot \log 2$$
$$= 2^{\log_2(\frac{x}{2})+1} \cdot \log 2 \qquad \dots[By (1)]$$
$$= 2^{\log_2(\frac{x}{2})+\log_2 2} \cdot \log 2$$
$$= 2^{\log_2(\frac{x}{2})+\log_2 2} \cdot \log 2$$
$$= 2^{\log_2(\frac{x}{2})+\log_2 2} = 2^{\log_2(\frac{x}{2})\log_2 2}$$
$$= x \log 2 \qquad \dots[\because a^{\log_3 x} = x]$$
$$\therefore \frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$$
$$= \frac{1}{x \log 2}.$$

Exercise 1.2 | Q 2.1 | Page 29



Find the derivative of the inverse function of the following : $y = x^2 \cdot e^x$

SOLUTION

 $y = x^{2} \cdot e^{x}$ Differentiating w.r.t. x, we get $\frac{dy}{dx} = \frac{d}{dx} (x^{2} \cdot e^{x})$ $= x^{2} \frac{d}{dx} (e^{x}) + e^{x} \frac{d}{dx} (x^{2})$ $= x^{2} \cdot e^{x} + e^{x} \times 2x$ $= xe^{x} (x + 2)$ The derivative of inverse function of y = f(x) is given by $\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$ $= \frac{1}{xe^{x}(x + 2)}.$

Exercise 1.2 | Q 2.2 | Page 29

Find the derivative of the inverse function of the following : $y = x \cos x$

SOLUTION

y = x cos x
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x cos x)$$

$$= x \frac{d}{dx}(cos x) + cos x \frac{d}{dx}(x)$$

$$= x(-sinx) + cosx x 1$$

$$= cosx - x sinx$$
The derivative of inverse function of y = f(x) is given by





$$\frac{\mathrm{dx}}{\mathrm{dy}} = \frac{1}{\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)}$$
$$= \frac{1}{\cos x - x \sin x}.$$

Exercise 1.2 | Q 2.3 | Page 29

Find the derivative of the inverse function of the following : $y = x \cdot 7^x$

SOLUTION

 $y = x \cdot 7^{x}$ Differentiating w.r.t. x, we get $\frac{dy}{dx} = \frac{d}{dx} (x \cdot 7^{x})$ $= x \frac{d}{dx} (7^{x}) + 7^{x} \frac{d}{dx} (x)$ $= x \cdot 7^{x} \log 7 + 7^{x} \times 1$ $= 7x (x \log 7 + 1)$ The derivative of inverse function of y = f(x) is given by $\frac{dy}{dx} = \frac{1}{\left(\frac{dy}{dx}\right)}$ $= \frac{1}{7^{x} (x \log 7 + 1)}.$

Exercise 1.2 | Q 2.4 | Page 29

Find the derivative of the inverse function of the following : $y = x^2 + \log x$





$$y = x^{2} + \log x$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx} (x^{2} + \log x)$$

$$= \frac{d}{dx} (x^{2}) + \frac{d}{dx} (\log x)$$

$$= 2x + \frac{1}{x}$$

$$= \frac{2x^{2} + 1}{x}$$

The derivative of inverse function of y = f(x) is given

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$$

$$= \frac{1}{\left(\frac{2x^{2} + 1}{x}\right)}$$

$$= \frac{x}{2x^{2} + 1}.$$

Exercise 1.2 | Q 2.5 | Page 29

Find the derivative of the inverse function of the following : $y = x \log x$

SOLUTION

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$$y = x \log x$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x \log x)$$

$$= x \frac{d}{dx}(\log x) + (\log x) \cdot \frac{d}{dx}(x)$$

$$= x \times \frac{1}{x} + (\log x) \times 1$$

$$= 1 + \log x$$

The derivative of inverse function of y = f(x) is given by

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$$

$$= \frac{1}{1 + \log x}.$$

Exercise 1.2 | Q 3.1 | Page 29

Find the derivative of the inverse of the following functions, and also find their value at the points indicated against them. $y = x^5 + 2x^3 + 3x$, at x = 1

SOLUTION





$$y = x^{5} + 2x^{3} + 3x$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx} (x^{5} + 2x^{3} + 3x)$$

$$= 5x^{4} + 2x 3x^{2} + 3x 1$$

$$= 5x^{4} + 6x^{2} + 3$$
The derivative of inverse function of y = f(x) is given

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$$

$$= \frac{1}{5x^{4} + 6x^{2} + 3}$$
At x = 1, $\frac{dx}{dy}$

$$= \frac{1}{(5x^{4} + 6x^{2} + 3)_{at x = 1}}$$

$$= \frac{1}{5(1)^{4} + 6(1)^{2} + 3}$$

$$= \frac{1}{5 + 6 + 3}$$

$$= \frac{1}{14}.$$

Exercise 1.2 | Q 3.2 | Page 29

Find the derivative of the inverse of the following functions, and also find their value at the points indicated against them. $y = e^x + 3x + 2$

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 $y = e^{x} + 3x + 2$ Differentiating w.r.t. x, we get $\frac{dy}{dx}(e^{x} + 3x + 2)$ $= e^{x} + 3 \times 1 + 0$ $= e^{x} + 3$ The derivative of inverse function of y = f(x) is given by

$$\frac{\mathrm{dx}}{\mathrm{dy}} = \frac{1}{\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)}$$
$$= \frac{1}{e^x + 3}$$
$$\operatorname{At x} = 0, \frac{\mathrm{dx}}{\mathrm{dy}}$$
$$= \frac{1}{(e^x + 3)_{atx=0}}$$
$$= \frac{1}{e^0 + 3}$$
$$= \frac{1}{\frac{1}{1+3}}$$
$$= \frac{1}{\frac{1}{4}}.$$

Exercise 1.2 | Q 3.3 | Page 29

Find the derivative of the inverse of the following functions, and also find their value at the points indicated against them. $y = 3x^2 + 2\log x^3$





$$y = 3x^{2} + 2\log x^{3}$$

$$= 3x^{2} + 6\log x$$
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{d}{dx} (3x^{2} + 6\log x)$$

$$= 3 \times 2x + 6 \times \frac{1}{x}$$

$$= 6x + \frac{6}{x}$$

$$= \frac{6x^{2} + 6}{x}$$
The derivative of inverse function of y = f(x) is given by
$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$$

$$= \frac{1}{\left(\frac{6x^{2} + 6}{x}\right)}$$

$$= \frac{x}{6x^{2} + 6}$$
At x = 1, $\frac{dx}{dy}$

$$= \left(\frac{x}{6x^{2} + 6}\right)_{at x = 1}$$

$$= \frac{1}{6(1)^{2} + 6}$$

$$= \frac{1}{12}.$$

Exercise 1.2 | Q 3.4 | Page 29

CLICK HERE



Find the derivative of the inverse of the following functions, and also find their value at the points indicated against them. $y = sin(x - 2) + x^2$

SOLUTION

$$y = \sin(x - 2) + x^{2}$$
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{d}{dx} [\sin(x - 2) + x^{2}]$$

$$= \frac{d}{dx} [\sin(x - 2)] + \frac{d}{dx} (x^{2})$$

$$= \cos(x - 2) \cdot \frac{d}{dx} (x - 2) + 2x$$

$$= \cos(x - 2) \cdot (1 - 0) + 2x$$

$$= \cos(x - 2) + 2x$$
The derivative of inverse function of y = f(x) is given by
$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$$

$$= \frac{1}{\cos(x - 2) + 2x}$$
At x = 2, $\frac{dx}{dy}$

$$= \left(\frac{1}{[\cos(x - 2) + 2x]}\right)_{\text{at } x = 2}$$

$$= \frac{1}{1 + 4}$$

$$= \frac{1}{5}.$$

Exercise 1.2 | Q 4 | Page 29 If $f(x) = x^3 + x - 2$, find $(f^{-1})'(-2)$.



 $f(x) = x^{3} + x - 2 \qquad ...(1)$ Differentiating w.r.t. x, we get $f'(x) = \frac{d}{dx} (x^{3} + x - 2)$ $= 3x^{2} + 1 = 0$ $= 3x^{2} + 1$ We know that $(f^{-1})'(y) = \frac{1}{f'(x)} \qquad ...(2)$ From (1), y = f(x) = -2, when x = 0 ∴ from (2), (f^{-1})'(-2) $= \frac{1}{f'(0)}$ $= \frac{1}{(3x^{2} + 1)_{atx=0}}$ $= \frac{1}{3(0) + 1}$ = 1.

Exercise 1.2 | Q 5.1 | Page 29

Using derivative, prove that: $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$





Let $f(x) = \tan^{-1}x + \cot^{-1}x$...(1) Differentiating w.r.t. x, we get $f'(x) = \frac{d}{dx} (\tan^{-1} x + \cot^{-1} x)$ $= \frac{\mathrm{d}}{\mathrm{d}\mathbf{x}} \left(\tan^{-1} x \right) + \frac{\mathrm{d}}{\mathrm{d}\mathbf{x}} \left(\cot^{-1} x \right)$ $=\frac{1}{1+x^2}-\frac{1}{1+x^2}$ = 0Since f'(x) = 0, f(x) is a constant function. Let f(x) = k. For any value of x, f(x) = kLet x = 0. Then f(0) = k...(2) From (1), $f(0) = \tan^{-1}(0) + \cot^{-1}(0)$ $= 0 + \frac{\pi}{2} = \frac{\pi}{2}$ $\therefore k = \frac{\pi}{2}$...[By (2)] $\therefore f(x) = k = \frac{\pi}{2}$ Hence, $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$[By (1)]

Exercise 1.2 | Q 5.2 | Page 29

Using derivative, prove that: $\sec^{-1}x + \csc^{-1}x = \frac{\pi}{2}$...[for $|x| \ge 1$]





Let
$$f(x) = \sec^{-1}x + \csc^{-1}x$$
 for $|x| \ge 1$...(1)
Differentiating w.r.t. x, we get
 $f'(x) = \frac{d}{dx} (\sec^{-1}x + \csc^{-1}x)$
 $= \frac{d}{dx} (\sec^{-1}x) + \frac{d}{dx} (\csc^{-1}x)$
 $= \frac{1}{x\sqrt{x^2 - 1}} - \frac{1}{x\sqrt{x^2 - 1}}$
 $= 0$
Since, $f'(x) = 0$, $f(x)$ is a constant function.
Let $f(x) = k$.
For any value of $x, f(x) = k$, where $|x| > 1$
Let $x = 2$.
Then, $f(2) = k$...(2)
From (1), $f(2) = \sec^{-1}(2) + \csc^{-1}(2)$
 $= \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$
 $\therefore k = \frac{\pi}{2}$ [By (2)]
 $\therefore f(x) = k = \frac{\pi}{2}$
Hence, $\sec^{-1}x + \csc^{-1}x = \frac{\pi}{2}$[By (1)]

Exercise 1.2 | Q 6.01 | Page 29

Differentiate the following w.r.t. $x : tan^{-1}(log x)$





Let
$$y = \tan^{-1}(\log x)$$

Differentiating w.r.t. x , we get
 $\frac{dy}{dx} = \frac{d}{dx} [\tan^{-1}(\log x)]$
 $= \frac{1}{1 + (\log x)^2} \cdot \frac{d}{dx} (\log x)$
 $= \frac{1}{1 + (\log x)^2} \times \frac{1}{x}$
 $= \frac{1}{x [1 + (\log x)^2]}$.

Exercise 1.2 | Q 6.02 | Page 29

Differentiate the following w.r.t. $x : cosec^{-1} (e^{-x})$

SOLUTION

Let
$$y = \operatorname{cosec}^{-1} (e^{-x})$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} [\operatorname{cosec}^{-1} (e^{-x})]$
 $= \frac{-1}{e^{-x} \sqrt{(e^{-x})^2 - 1}} \cdot \frac{d}{dx} (e^{-x})$
 $= \frac{1}{e^{-x} \sqrt{e^{-2}x - 1}} \times e^{-x} \cdot \frac{d}{dx} (-x)$
 $= \frac{-1}{\sqrt{e^{-2}x - 1}} \times -1$
 $= \frac{1}{\sqrt{\frac{1}{e^{2x}} - 1}}$

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$$= \frac{e^x}{\sqrt{1 - e^2x}}.$$

Exercise 1.2 | Q 6.03 | Page 29

Differentiate the following w.r.t. $x : \cot^{-1}(x^3)$

SOLUTION

Let
$$y = \cot^{-1}(x^3)$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx} \left[\cot^{-1}(x^3) \right]$$

$$= \frac{-1}{1 + (x^3)^2} \cdot \frac{d}{dx} (x^3)$$

$$= \frac{-1}{1 + x^6} \times 3x^2$$

$$= \frac{-3x^2}{1 + x^6}.$$

Exercise 1.2 | Q 6.04 | Page 29 Differentiate the following w.r.t. $x : \cot^{-1}(4^x)$

SOLUTION

Let $y = \cot^{-1}(4^x)$ Differentiating w.r.t. x, we get $\frac{dy}{dx} = \frac{d}{dx} \left[\cot^{-1}(4^x) \right]$ $= \frac{-1}{1 + (4^x)^2} \cdot \frac{d}{dx} (4^x)$ $= \frac{-1}{1 + 4^2 x} \times 4^x \log 4$ $= \frac{4^x \log 4}{1 + 4^2 x}.$

Exercise 1.2 | Q 6.05 | Page 29

Differentiate the following w.r.t. x : $\tan^{-1}(\sqrt{x})$

SOLUTION

Let
$$y = \tan^{-1}(\sqrt{x})$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} [\tan^{-1}(\sqrt{x})]$
 $= \frac{1}{1 + (\sqrt{x})^2} \cdot \frac{d}{dx} (\sqrt{x})$
 $= \frac{1}{1 + x} \times \frac{1}{2\sqrt{x}}$
 $= \frac{1}{2\sqrt{x}(1 + x)}.$

Exercise 1.2 | Q 6.06 | Page 29

Differentiate the following w.r.t. x : $\sin^{-1}\left(\sqrt{rac{1+x^2}{2}}
ight)$





Let
$$y = \sin^{-1}\left(\sqrt{\frac{1+x^2}{2}}\right)$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}\left[\sin^{-1}\left(\sqrt{\frac{1+x^2}{2}}\right)\right]$
 $= \frac{1}{\sqrt{1-\left(\sqrt{\frac{1+x^2}{2}}\right)^2}} \cdot \frac{d}{dx}\left(\sqrt{\frac{1+x^2}{2}}\right)$
 $= \frac{1}{\sqrt{\left(1-\frac{1+x^2}{2}\right)}} \times \frac{1}{\sqrt{2}} \frac{d}{dx}\left(\sqrt{1+x^2}\right)$
 $= \frac{\sqrt{2}}{\sqrt{2}-1-x^2} \times \frac{1}{\sqrt{2}} \times \frac{1}{2\sqrt{1+x^2}} \cdot \frac{d}{dx}(1+x^2)$
 $= \frac{1}{\sqrt{1-x^2}} \times \frac{1}{2\sqrt{1+x^2}} \cdot (0+2x)$
 $= \frac{x}{\sqrt{(1-x^2)(1+x^2)}}$
 $= \frac{x}{\sqrt{1-x^4}}.$

Exercise 1.2 | Q 6.07 | Page 29

Differentiate the following w.r.t. $x : \cos^{-1}(1 - x^2)$





Let
$$y = \cos^{-1}(1 - x^2)$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} \left[\cos^{-1} (1 - x^2) \right]$
 $= \frac{-1}{\sqrt{1 - (1 - x^2)^2}} \cdot \frac{d}{dx} (1 - x^2)$
 $= \frac{-1}{\sqrt{1 - (1 - 2x^2 + x^4)}} \cdot (0 - 2x)$
 $= \frac{2x}{\sqrt{2x^2 - x^4}}$
 $= \frac{2x}{x\sqrt{2 - x^2}}$
 $= \frac{2}{\sqrt{2 - x^2}}$.

Exercise 1.2 | Q 6.08 | Page 29

Differentiate the following w.r.t. x : $\sin^{-1}\left(x^{rac{3}{2}}
ight)$





Let
$$y = \sin^{-1}\left(x^{\frac{3}{2}}\right)$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}\left[\sin^{-1}\left(x^{\frac{3}{2}}\right)\right]$
 $= \frac{1}{\sqrt{1-\left(x^{\frac{3}{2}}\right)^2}} \cdot \frac{d}{dx}\left(x^{\frac{3}{2}}\right)$
 $= \frac{1}{\sqrt{1-x^3}} \times \frac{3}{2}x^{\frac{3}{2}}$
 $= \frac{3\sqrt{x}}{2\sqrt{1-x^3}}.$

Exercise 1.2 | Q 6.09 | Page 29

Differentiate the following w.r.t. $x : \cos^{3}[\cos^{-1}(x^{3})]$

SOLUTION

Let
$$y = \cos^{3}[\cos^{-1}(x^{3})]$$

= $[\cos(\cos^{-1}x^{3})]^{3}$
= $(x^{3})^{3}$
= x^{9}
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}(x^{9})$
= $9x^{8}$.

Exercise 1.2 | Q 6.1 | Page 29 Differentiate the following w.r.t. x : $\sin^4 \left[\sin^{-1} \left(\sqrt{x} \right) \right]$





Let
$$y = \sin^4 [\sin^{-1}(\sqrt{x})]$$

 $= \{ \sin [\sin^{-1}(\sqrt{x})] \}^4$
 $= (\sqrt{x})^4$
 $= x^2$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}(x^2)$
 $= 2x.$

Exercise 1.2 | Q 6.02 | Page 29

Differentiate the following w.r.t. $x : cosec^{-1} (e^{-x})$

SOLUTION

Let
$$y = \operatorname{cosec}^{-1} (e^{-x})$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} [\operatorname{cosec}^{-1} (e^{-x})]$
 $= \frac{-1}{e^{-x} \sqrt{(e^{-x})^2 - 1}} \cdot \frac{d}{dx} (e^{-x})$
 $= \frac{1}{e^{-x} \sqrt{e^{-2}x - 1}} \times e^{-x} \cdot \frac{d}{dx} (-x)$
 $= \frac{-1}{\sqrt{e^{-2}x - 1}} \times -1$
 $= \frac{1}{\sqrt{\frac{1}{e^{2x}} - 1}}$
 $= \frac{e^x}{\sqrt{1 - e^2x}}.$

Exercise 1.2 | Q 6.03 | Page 29 Differentiate the following w.r.t. $x : \cot^{-1}(x^3)$

SOLUTION

Let
$$y = \cot^{-1}(x^3)$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx} \left[\cot^{-1}(x^3) \right]$$

$$= \frac{-1}{1 + (x^3)^2} \cdot \frac{d}{dx} (x^3)$$

$$= \frac{-1}{1 + x^6} \times 3x^2$$

$$= \frac{-3x^2}{1 + x^6}.$$

Exercise 1.2 | Q 6.04 | Page 29

Differentiate the following w.r.t. $x : \cot^{-1}(4^x)$

SOLUTION

Let
$$y = \cot^{-1}(4^{x})$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx} \left[\cot^{-1}(4^{x}) \right]$$

$$= \frac{-1}{1 + (4^{x})^{2}} \cdot \frac{d}{dx} (4^{x})$$

$$= \frac{-1}{1 + 4^{2}x} \times 4^{x} \log 4$$

$$= \frac{4^{x} \log 4}{1 + 4^{2}x}.$$



Exercise 1.2 | Q 6.05 | Page 29

Differentiate the following w.r.t. x : $an^{-1}(\sqrt{x})$

SOLUTION

Let
$$y = \tan^{-1}(\sqrt{x})$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} [\tan^{-1}(\sqrt{x})]$
 $= \frac{1}{1 + (\sqrt{x})^2} \cdot \frac{d}{dx} (\sqrt{x})$
 $= \frac{1}{1 + x} \times \frac{1}{2\sqrt{x}}$
 $= \frac{1}{2\sqrt{x}(1 + x)}.$

Exercise 1.2 | Q 6.06 | Page 29

Differentiate the following w.r.t. x : $\sin^{-1}\left(\sqrt{rac{1+x^2}{2}}
ight)$

SOLUTION

Let
$$y = \sin^{-1}\left(\sqrt{\frac{1+x^2}{2}}\right)$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}\left[\sin^{-1}\left(\sqrt{\frac{1+x^2}{2}}\right)\right]$
 $= \frac{1}{\sqrt{1-\left(\sqrt{\frac{1+x^2}{2}}\right)^2}} \cdot \frac{d}{dx}\left(\sqrt{\frac{1+x^2}{2}}\right)$


$$= \frac{1}{\sqrt{\left(1 - \frac{1 + x^2}{2}\right)}} \times \frac{1}{\sqrt{2}} \frac{d}{dx} \left(\sqrt{1 + x^2}\right)$$
$$= \frac{\sqrt{2}}{\sqrt{2} - 1 - x^2} \times \frac{1}{\sqrt{2}} \times \frac{1}{2\sqrt{1 + x^2}} \cdot \frac{d}{dx} \left(1 + x^2\right)$$
$$= \frac{1}{\sqrt{1 - x^2}} \times \frac{1}{2\sqrt{1 + x^2}} \cdot \left(0 + 2x\right)$$
$$= \frac{x}{\sqrt{(1 - x^2)(1 + x^2)}}$$

Exercise 1.2 | Q 6.07 | Page 29

Differentiate the following w.r.t. $x : \cos^{-1}(1 - x^2)$

SOLUTION

Let
$$y = \cos^{-1}(1 - x^2)$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} [\cos^{-1}(1 - x^2)]$
 $= \frac{-1}{\sqrt{1 - (1 - x^2)^2}} \cdot \frac{d}{dx} (1 - x^2)$
 $= \frac{-1}{\sqrt{1 - (1 - 2x^2 + x^4)}} \cdot (0 - 2x)$
 $= \frac{2x}{\sqrt{2x^2 - x^4}}$
 $= \frac{2x}{x\sqrt{2 - x^2}}$
 $= \frac{2}{\sqrt{2 - x^2}}$.

Exercise 1.2 | Q 6.08 | Page 29

Differentiate the following w.r.t. x : $\sin^{-1}\left(x^{\frac{3}{2}}\right)$

SOLUTION

Let
$$y = \sin^{-1}\left(x^{\frac{3}{2}}\right)$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}\left[\sin^{-1}\left(x^{\frac{3}{2}}\right)\right]$
 $= \frac{1}{\sqrt{1-\left(x^{\frac{3}{2}}\right)^2}} \cdot \frac{d}{dx}\left(x^{\frac{3}{2}}\right)$
 $= \frac{1}{\sqrt{1-x^3}} \times \frac{3}{2}x^{\frac{3}{2}}$
 $= \frac{3\sqrt{x}}{2\sqrt{1-x^3}}.$

Exercise 1.2 | Q 6.09 | Page 29 Differentiate the following w.r.t. $x : \cos^{3}[\cos^{-1}(x^{3})]$

SOLUTION

Let
$$y = \cos^{3}[\cos^{-1}(x^{3})]$$

= $[\cos(\cos^{-1}x^{3})]^{3}$
= $(x^{3})^{3}$
= x^{9}
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}(x^{9})$
= $9x^{8}$.
Exercise 1.2 | Q 6.1 | Page 29
Differentiate the following w.r.t. x : $\sin^{4}[\sin^{-1}(\sqrt{x})]$





Exercise 1.2 | Q 6.1 | Page 29

Differentiate the following w.r.t. x : $\sin^4 [\sin^{-1} (\sqrt{x})]$

SOLUTION

Let
$$y = \sin^4 [\sin^{-1}(\sqrt{x})]$$

 $= \{ \sin [\sin^{-1}(\sqrt{x})] \}^4$
 $= (\sqrt{x})^4$
 $= x^2$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} (x^2)$

$$d\mathbf{x} = 2\mathbf{x}.$$

Exercise 1.2 | Q 7.01 | Page 29

Differentiate the following w.r.t. x : $\cot^{-1} \left[\cot \left(e^{x^2} \right)
ight]$

SOLUTION

Let
$$y = \cot^{-1} \left[\cot \left(e^{x^2} \right) \right] = e^{x^2}$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} \left(e^{x^2} \right)$
 $= e^{x^2} \cdot \frac{d}{dx} \left(x^2 \right)$
 $= e^{x^2} \times 2x$
 $= 2xe^2$.

Exercise 1.2 | Q 7.02 | Page 29

Differentiate the following w.r.t. x : $\csc^{-1}\left[\frac{1}{\cos(5^x)}\right]$





Let
$$y = \csc^{-1} \left[\frac{1}{\cos(5^x)} \right]$$

= $\csc^{-1}[\sec(5^x)]$
= $\csc^{-1}\left[\csc\left(\frac{\pi}{2} - 5^x\right)\right]$
= $\frac{\pi}{2} - 5^x$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2} - 5^x\right)$
= $\frac{d}{dx}\left(\frac{\pi}{2}\right) - \frac{d}{dx}(5^x)$
= $0 - 5^x \cdot \log 5$
= $-5^x \cdot \log 5$.

Exercise 1.2 | Q 7.03 | Page 29

Differentiate the following w.r.t. x : $\cos^{-1}\left(\sqrt{rac{1+\cos x}{2}}
ight)$





Let
$$y = \cos^{-1}\left(\sqrt{\frac{1+\cos x}{2}}\right)$$

 $= \cos^{-1}\left(\sqrt{\frac{2\cos^2\left(\frac{x}{2}\right)}{2}}\right)$
 $= \cos^{-1}\left[\cos\left(\frac{x}{2}\right)\right]$
 $= \frac{x}{2}$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{2}\right)$
 $= \frac{1}{2}\frac{d}{dx}(x)$
 $= \frac{1}{2} \times 1$
 $= \frac{1}{2}$.

Exercise 1.2 | Q 7.04 | Page 29

Differentiate the following w.r.t. x : $\cos^{-1} \Biggl(rac{\sqrt{1-\cos(x^2)}}{2} \Biggr)$





Let
$$y = \cos^{-1}\left(\frac{\sqrt{1-\cos(x^2)}}{2}\right)$$

$$= \cos^{-1}\left(\sqrt{\frac{2\sin^2\left(\frac{x^2}{2}\right)}{2}}\right)$$

$$= \cos^{-1}\left[\sin\left(\frac{x^2}{2}\right)\right]$$

$$= \cos^{-1}\left[\cos\left(\frac{\pi}{2} - \frac{x^2}{2}\right)\right]$$

$$= \frac{\pi}{2} - \frac{x^2}{2}$$
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2} - \frac{x^2}{2}\right)$$

$$= \frac{d}{dx}\left(\frac{\pi}{2}\right) - \frac{1}{2}\frac{d}{dx}(x^2)$$

$$= 0 - \frac{1}{2} \times 2x$$

$$= -x.$$

Exercise 1.2 | Q 7.05 | Page 29

Differentiate the following w.r.t. x :
$$\tan^{-1}\left[rac{1- an\left(rac{x}{2}
ight)}{1+ an\left(rac{x}{2}
ight)}
ight]$$





Let
$$y = \tan^{-1} \left[\frac{1 - \tan\left(\frac{x}{2}\right)}{1 + \tan\left(\frac{x}{2}\right)} \right]$$

$$= \tan^{-1} \left[\frac{\tan\left(\frac{\pi}{4}\right) - \tan\left(\frac{x}{2}\right)}{1 + \tan\left(\frac{\pi}{4}\right) \cdot \tan\left(\frac{x}{2}\right)} \right] \dots \left[\because \tan \frac{\pi}{4} = 1 \right]$$

$$= \tan^{-1} \left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right) \right]$$

$$= \frac{\pi}{4} - \frac{x}{2}$$
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$= \frac{d}{dx} \left(\frac{\pi}{4}\right) - \frac{1}{2} \frac{d}{dx}(x)$$

$$= 0 - \frac{1}{2} \times 1$$

$$= -\frac{1}{2}.$$

Exercise 1.2 | Q 7.06 | Page 29

Differentiate the following w.r.t. x : $\csc^{-1} igg(\frac{1}{4\cos^3 2x - 3\cos 2x} igg)$





Let
$$y = \csc^{-1}\left(\frac{1}{4\cos^3 2x - 3\cos 2x}\right)$$

$$= \csc^{-1}\left(\frac{1}{\cos 6x}\right) \quad ...[\because \cos 3x = 4\cos^3 x - 3\cos x]$$

$$= \csc^{-1}(\sec 6x)$$

$$= \csc^{-1}\left[\csc\left(\frac{\pi}{2} - 6x\right)\right]$$

$$= \frac{\pi}{2} - 6x$$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2} - 6x\right)$$

$$= \frac{d}{dx}\left(\frac{\pi}{2}\right) - 6\frac{d}{dx}(x)$$

$$= 0 - 6 \times 1$$

$$= -6$$

Exercise 1.2 | Q 7.07 | Page 29

Differentiate the following w.r.t. $x : tan^{-1} \left[\frac{1 + cos(\frac{x}{3})}{sin(\frac{x}{3})} \right]$





Let
$$y = \tan^{-1} \left[\frac{1 + \cos\left(\frac{x}{3}\right)}{\sin\left(\frac{x}{3}\right)} \right]$$

 $= \tan^{-1} \left[\frac{2\cos^2\left(\frac{x}{6}\right)}{2\sin\left(\frac{x}{6}\right)\cos\left(\frac{x}{6}\right)} \right]$
 $= \tan^{-1} \left[\cot\left(\frac{x}{6}\right) \right]$
 $= \tan^{-1} \left[\tan\left(\frac{\pi}{2} - \frac{x}{6}\right) \right]$
 $= \frac{\pi}{2} - \frac{x}{6}$
Differentiating w.r.t. x, we get

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x} \left(\frac{\pi}{2} - \frac{x}{6}\right)$$
$$= \frac{\mathrm{d}}{\mathrm{d}x} \left(\frac{\pi}{2}\right) - \frac{1}{6} \frac{\mathrm{d}}{\mathrm{d}x}(x)$$
$$= 0 - \frac{1}{6} \times 1$$
$$= -\frac{1}{6}.$$

Exercise 1.2 | Q 7.08 | Page 29

Differentiate the following w.r.t. x :
$$\cot^{-1}\left(\frac{\sin 3x}{1+\cos 3x}\right)$$





Let
$$y = \cot^{-1}\left(\frac{\sin 3x}{1 + \cos 3x}\right)$$

$$= \cot^{-1}\left[\frac{2\sin\left(\frac{3x}{2}\right)\cos\left(\frac{3x}{2}\right)}{2\cos^{2}\left(\frac{3x}{2}\right)}\right]$$

$$= \cot^{-1}\left[\tan\left(\frac{3x}{2}\right)\right]$$

$$= \cot^{-1}\left[\cot\left(\frac{\pi}{2} - \frac{3x}{2}\right)\right]$$

$$= \frac{\pi}{2} - \frac{3x}{2}$$
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2} - \frac{3x}{2}\right)$$

$$= \frac{d}{dx}\left(\frac{\pi}{2}\right) - \frac{3}{2}\frac{dx}{dx}$$

$$= 0 - \frac{3}{2} \times 1$$

$$= -\frac{3}{2}.$$

Exercise 1.2 | Q 7.09 | Page 30

Differentiate the following w.r.t. x :
$$an^{-1} igg(rac{\cos 7x}{1+\sin 7x} igg)$$

SOLUTION





Let
$$y = \tan^{-1}\left(\frac{\cos 7x}{1 + \sin 7x}\right)$$

$$= \tan^{-1}\left[\frac{\sin(\frac{\pi}{2} - 7x)}{1 + \cos(\frac{\pi}{2} - 7x)}\right]$$

$$= \tan^{-1}\left[\frac{2\sin(\frac{\pi}{4} - \frac{7x}{2}) \cdot \cos(\frac{\pi}{4} - \frac{7x}{2})}{2\cos^2(\frac{\pi}{4} - \frac{7x}{2})}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{7x}{2}\right)\right]$$

$$= \frac{\pi}{4} - \frac{7x}{2}$$
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{4} - \frac{7x}{2}\right)$$

$$= \frac{d}{dx}\left(\frac{\pi}{4}\right) - \frac{7}{2}\frac{d}{dx}(x)$$

$$= 0 - \frac{7}{2} \times 1$$

$$= -\frac{7}{2}.$$

Exercise 1.2 | Q 7.1 | Page 30

Differentiate the following w.r.t. x : $an^{-1}\left(\sqrt{rac{1+\cos x}{1-\cos x}}
ight)$





Let
$$y = \tan^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$$

$$= \tan^{-1}\left[\sqrt{\frac{2\cos^2\left(\frac{x}{2}\right)}{2\sin^2\left(\frac{x}{2}\right)}}\right]$$

$$= \tan^{-1}\left[\cot\left(\frac{x}{2}\right)\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right]$$

$$= \frac{\pi}{2} - \frac{\pi}{2}$$
Differentiate the following w.r.t.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2} - \frac{x}{2}\right)$$

$$= \frac{d}{dx}\left(\frac{\pi}{2}\right) - \frac{1}{2}\frac{d}{dx}(x)$$

$$= 0 - \frac{1}{2} \times 1$$

$$= -\frac{1}{2}.$$

Exercise 1.2 | Q 7.11 | Page 30

Differentiate the following w.r.t. $x : \tan^{-1} (\operatorname{cosec} x + \operatorname{cot} x)$

x :





Let
$$y = \tan^{-1} (\operatorname{cosec} x + \cot x)$$

$$= \tan^{-1} \left(\frac{1}{\sin x} + \frac{\cos x}{\sin x} \right)$$

$$= \tan^{-1} \left(\frac{1 + \cos x}{\sin x} \right)$$

$$= \tan^{-1} \left[\frac{2 \cos^2 \left(\frac{x}{2} \right)}{2 \sin \left(\frac{x}{2} \right) \cdot \cos \left(\frac{x}{2} \right)} \right]$$

$$= \tan^{-1} \left[\cot \left(\frac{x}{2} \right) \right]$$

$$= \tan^{-1} \left[\tan \left(\frac{\pi}{2} - \frac{x}{2} \right) \right]$$

$$= \frac{\pi}{2} - \frac{x}{2}$$
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{\pi}{2} - \frac{x}{2} \right)$$

$$= \frac{d}{dx} \left(\frac{\pi}{2} \right) - \frac{1}{2} \frac{d}{dx} (x)$$

$$= 0 - \frac{1}{2} \times 1$$

$$= -\frac{1}{2}.$$

Exercise 1.2 | Q 7.12 | Page 30

Differentiate the following w.r.t. x :
$$\cot^{-1}$$

$$\left[\frac{\sqrt{1+\sin\left(\frac{4}{3}\right)}+\sqrt{1-\sin\left(\frac{4x}{3}\right)}}{\sqrt{1+\sin\left(\frac{4x}{3}\right)}-\sqrt{1-\sin\left(\frac{4x}{3}\right)}}\right]$$

г





Let
$$y = \cot^{-1}\left[\frac{\sqrt{1+\sin\left(\frac{4}{3}\right)} + \sqrt{1-\sin\left(\frac{4x}{3}\right)}}{\sqrt{1+\sin\left(\frac{4x}{3}\right)} - \sqrt{1-\sin\left(\frac{4x}{3}\right)}}\right]$$

$$= 1 + \sin\left(\frac{4x}{3}\right)$$

$$= 1 + \cos\left(\frac{\pi}{2} - \frac{4x}{3}\right)$$

$$= 2\cos^{2}\left(\frac{\pi}{4} - \frac{2x}{3}\right)$$

$$\therefore \sqrt{1 + \sin\left(\frac{4x}{3}\right)} = \sqrt{2}\cos\left(\frac{\pi}{4} - \frac{2x}{3}\right)$$
Also, $1 - \sin\left(\frac{4x}{3}\right)$

$$= 1 - \cos\left(\frac{\pi}{2} - \frac{4x}{3}\right)$$

$$= 2\sin^{2}\left(\frac{\pi}{4} - \frac{2x}{3}\right)$$

$$= 2\sin^{2}\left(\frac{\pi}{4} - \frac{2x}{3}\right)$$

$$\therefore \sqrt{1 - \sin\left(\frac{4x}{3}\right)} = \sqrt{2}\sin\left(\frac{\pi}{4} - \frac{2x}{3}\right)$$

$$\therefore \sqrt{1 + \sin\left(\frac{4x}{3}\right)} + \sqrt{1 - \sin\left(\frac{4x}{3}\right)}$$

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$$\frac{1}{\sqrt{1 + \sin(\frac{4x}{3}) - \sqrt{1 - \sin(\frac{4x}{3})}} }{\sqrt{2}\cos(\frac{\pi}{4} - \frac{2x}{3}) + \sqrt{2}\sin(\frac{\pi}{4} - \frac{2x}{3})}$$

$$= \frac{\sqrt{2}\cos(\frac{\pi}{4} - \frac{2x}{3}) + \sin(\frac{\pi}{4} - \frac{2x}{3})}{\sqrt{2}\cos(\frac{\pi}{4} - \frac{2x}{3}) - \sqrt{2}\sin(\frac{\pi}{4} - \frac{2x}{3})}$$

$$= \frac{\cos(\frac{\pi}{4} - \frac{2x}{3}) - \sin(\frac{\pi}{4} - \frac{2x}{3})}{1 - \tan(\frac{\pi}{4} - \frac{2x}{3})} \dots \left[\text{Dividing by } \cos\left(\frac{\pi}{4} - \frac{2x}{3}\right) \right]$$

$$= \frac{\tan\frac{\pi}{4} + \tan(\frac{\pi}{4} - \frac{2x}{3})}{1 - \tan\frac{\pi}{4} \cdot \tan(\frac{\pi}{4} - \frac{2x}{3})} \dots \left[\because \tan\frac{\pi}{4} = 1 \right]$$

$$= \tan\left[\frac{\pi}{4} + \frac{\pi}{4} - \frac{2x}{3}\right]$$

$$= \tan\left(\frac{\pi}{2} - \frac{2x}{3}\right)$$

$$= \cot\left(\frac{2x}{3}\right)$$

$$\therefore y = \cot^{-1}\left[\cot\left(\frac{2x}{3}\right)\right] = \frac{2x}{3}$$

$$\text{Differentiating w.r.t. x, we get }$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x}{3}\right)$$

$$= \frac{2}{3}\frac{d}{dx}(x)$$

$$= \frac{2}{3} \times 1$$

$$= \frac{2}{3}.$$

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Exercise 1.2 | Q 8.1 | Page 30

Differentiate the following w.r.t. x : $\sin^{-1} \left(\frac{4 \sin x + 5 \cos x}{\sqrt{41}} \right)$

SOLUTION

Let
$$y = \sin^{-1}\left(\frac{4\sin x + 5\cos x}{\sqrt{41}}\right)$$

 $= \sin^{-1}\left[(\sin x)\left(\frac{4}{\sqrt{41}}\right) + (\cos x)\left(\frac{5}{\sqrt{41}}\right)\right]$
Since, $\left(\frac{4}{\sqrt{41}}\right)^2 + \left(\frac{5}{\sqrt{41}}\right)^2 = \frac{16}{41} + \frac{25}{41} = 1$,
we can write, $\frac{4}{\sqrt{41}} = \cos \infty$ and $\frac{5}{\sqrt{41}} = \sin \infty$.

$$\therefore y = \sin^{-1} (\sin x \cos \infty + \cos x \sin \infty)$$

= $\sin^{-1} [\sin(x + \infty)]$
= $x + \infty$, where ∞ is a constant
Differentiating w.r.t. x , we get
 $\frac{dy}{dx} = \frac{d}{dx} (x + \infty)$
= $\frac{d}{dx} (x) + \frac{d}{dx} (\infty)$
= $1 + 0$
= 1.

Exercise 1.2 | Q 8.2 | Page 30

Differentiate the following w.r.t. x :
$$\cos^{-1}\left(\frac{\sqrt{3}\cos x - \sin x}{2}\right)$$





Let
$$y = \cos^{-1}\left(\frac{\sqrt{3}\cos x - \sin x}{2}\right)$$

$$= \cos^{-1}\left[\left(\cos x\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\sin x\right)\left(\frac{1}{2}\right)\right]$$

$$= \cos^{-1}\left(\cos x \cos \frac{\pi}{6} - \sin x \sin \frac{\pi}{6}\right) \dots \left[\because \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}, \sin \frac{\pi}{6} = \frac{1}{2}\right]$$

$$= \cos^{-1}\left[\cos\left(x + \frac{\pi}{6}\right)\right]$$

$$= x + \frac{\pi}{6}$$
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{d}{dx}\left(x + \frac{\pi}{6}\right)$$

$$= \frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{\pi}{6}\right)$$

$$= 1 + 0$$

$$= 1.$$

Exercise 1.2 | Q 8.3 | Page 30

Differentiate the following w.r.t. x : $\sin^{-1} \left(rac{\cos \sqrt{x} + \sin \sqrt{x}}{\sqrt{2}}
ight)$





$$y = \sin^{-1}\left(\frac{\cos\sqrt{x} + \sin\sqrt{x}}{\sqrt{2}}\right)$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\cos\sqrt{x} + \frac{1}{\sqrt{2}}\sin\sqrt{x}\right)$$
Put,

$$\frac{1}{\sqrt{2}} = \sin x$$

$$\frac{1}{\sqrt{2}} = \cos \alpha$$
Also,

$$\sin^{2}\alpha + \cos^{2}\alpha = \left(\frac{1}{\sqrt{2}}\right)^{2} + \left(\frac{1}{\sqrt{2}}\right)^{2} = 1$$
And,

$$\tan \alpha = 1$$

$$\therefore \alpha = \tan^{-1}1$$

$$y = \sin^{-1}(\sin \alpha . \cos \sqrt{x} + \cos \alpha . \sin(x))$$

$$= \sin^{-1}(\sin(\alpha + \sqrt{x}))$$

$$y = \alpha + \sqrt{x}$$

$$y = \tan^{-1}(1) + \sqrt{x}$$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\tan^{-1} + \sqrt{x})$$

$$= 0 + \frac{1}{2\sqrt{x}}$$





Exercise 1.2 | Q 8.4 | Page 30

Differentiate the following w.r.t. x :
$$\cos^{-1} \left(rac{3\cos 3x - 4\sin 3x}{5}
ight)$$

SOLUTION

Let
$$y = \cos^{-1}\left(\frac{3\cos 3x - 4\sin 3x}{5}\right)$$

 $= \cos^{-1}\left[(\cos 3x)\left(\frac{3}{5}\right) - (\sin 3x)\left(\frac{4}{5}\right)\right]$
Since, $\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2$
 $= \frac{9}{25} + \frac{16}{25} = 1$
we can write, $\frac{3}{5} = \cos \infty$ and $\frac{4}{5} = \sin \infty$.
 $\therefore y = \cos^{-1}(\cos 3x \cos \infty - \sin 3x \sin \infty)$
 $= \cos^{-1}[\cos(3x + \infty)]$
 $= 3x + \infty$, where ∞ is a constant
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}(3x + \infty)$

$$= 3\frac{d}{dx}(x) + \frac{d}{dx}(\infty)$$
$$= 3 \times 1 + 0$$
$$= 3.$$

Exercise 1.2 | Q 8.5 | Page 30 Differentiate the following w.r.t. x : $\cos^{-1}\left[\frac{3\cos(e^x) + 2\sin(e^x)}{\sqrt{13}}\right]$



$$y = \cos^{-1}\left(\frac{3\cos(e^{x}) + 2\sin(e^{x})}{\sqrt{13}}\right)$$

$$= \cos^{-1}\left(\cos(e^{x}) \cdot \frac{3}{\sqrt{13}} + \sin(e^{x})\frac{2}{\sqrt{13}}\right)$$
Put,

$$\frac{3}{\sqrt{13}} = \cos x$$

$$\frac{2}{\sqrt{3}} = \sin x$$
Also,

$$\sin^{2}\alpha + \cos^{2}\alpha = \frac{9}{13} + \frac{4}{13} = 1$$
And,

$$\tan \alpha = \frac{\sin x}{\cos \alpha} = \frac{2}{3}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{2}{3}\right)$$

$$y = \cos^{-1}(\cos e^{x} \cdot \cos \alpha + \sin e^{x} \cdot \cos \alpha)$$

$$y = \cos^{-1}(\cos e^{x} - \alpha)) \qquad \because \cos^{-1}x \cdot (\cos x) = x$$

$$y = e^{x} - \alpha$$

$$= e^{x} = \tan^{-1}\left(\frac{2}{3}\right)$$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{x} - \tan^{-1}\left(\frac{2}{3}\right)\right)$$

$$= e^{x} - 0$$

$$= e^{x}.$$





Exercise 1.2 | Q 8.5 | Page 30

Differentiate the following w.r.t. x :
$$\cos^{-1}\left[\frac{3\cos(e^x) + 2\sin(e^x)}{\sqrt{13}}\right]$$

SOLUTION

$$y = \cos^{-1}\left(\frac{3\cos(e^{x}) + 2\sin(e^{x})}{\sqrt{13}}\right)$$

$$= \cos^{-1}\left(\cos(e^{x}) \cdot \frac{3}{\sqrt{13}} + \sin(e^{x})\frac{2}{\sqrt{13}}\right)$$
Put,

$$\frac{3}{\sqrt{13}} = \cos x$$

$$\frac{2}{\sqrt{3}} = \sin x$$
Also,

$$\sin^{2}\alpha + \cos^{2}\alpha = \frac{9}{13} + \frac{4}{13} = 1$$
And,

$$\tan \alpha = \frac{\sin x}{\cos \alpha} = \frac{2}{3}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{2}{3}\right)$$

$$y = \cos^{-1}(\cos e^{x} \cdot \cos \alpha + \sin e^{x} \cdot \cos \alpha)$$

$$y = \cos^{-1}(\cos e^{x} - \alpha)) \qquad \because \cos^{-1}x \cdot (\cos x) = x$$

$$y = e^{x} - \alpha$$

$$= e^{x} = \tan^{-1}\left(\frac{2}{3}\right)$$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{x} - \tan^{-1}\left(\frac{2}{3}\right)\right)$$

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$$= e^{x} - 0$$

 $= e^{x}$.

Exercise 1.2 | Q 8.6 | Page 30

Differentiate the following w.r.t. x : $\csc^{-1}\left[\frac{10}{6\sin(2^x)-8\cos(2^x)}
ight]$

SOLUTION

Let
$$y = \csc^{-1} \left[\frac{10}{6 \sin(2^x) - 8 \cos(2^x)} \right]$$

$$= \sin^{-1} \left[\frac{6 \sin(2^x) - 8 \cos(2^x)}{10} \right] \dots \left[\because \csc^{-1}x = \sin^{-1} \left(\frac{1}{x} \right) \right]$$

$$= \sin^{-1} \left[\left\{ \sin(2^x) \right\} \left(\frac{6}{10} \right) - \left\{ \cos(2^x) \right\} \left(\frac{8}{10} \right) \right]$$
Since, $\left(\frac{6}{10} \right)^2 + \left(\frac{8}{0} \right)^2 = \frac{36}{100} + \frac{64}{100} = 1$,
we can write, $\frac{6}{10} = \cos \infty$ and $\frac{8}{10} = \sin \infty$.
 $\therefore y = \sin^{-1} [\sin(2^x) \cdot \cos \infty - \cos(2^x) \cdot \sin \infty]$

$$= \sin^{-1} [\sin(2^x - \infty)]$$

$$= 2^x - \infty$$
, where ∞ is a constant
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} (2^x - \infty)$

$$= \frac{d}{dx} (2^x) - \frac{d}{dx} (\infty)$$

$$= 2^x \log 2 - 0$$

$$= 2^x \log 2.$$

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Exercise 1.2 | Q 9.01 | Page 30

Differentiate the following w.r.t. x : $\cos^{-1} igg(rac{1-x^2}{1+x^2} igg)$

SOLUTION

Let
$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

Put $x = \tan\theta$.
Then $\theta = \tan^{-1}x$
 $\therefore y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$
 $= \cos^{-1}(\cos 2\theta)$
 $= 2\theta$
 $= 2\tan^{-1}x$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}(2\tan^{-1}x)$
 $= 2\frac{d}{dx}(\tan^{-1}x)$
 $= 2 \times \frac{1}{1+x^2}$
 $= \frac{2}{1+x^2}$.

Exercise 1.2 | Q 9.02 | Page 30

Differentiate the following w.r.t. x : $an^{-1} igg(rac{2x}{1-x^2} igg)$





Let
$$y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Put $x = \tan\theta$.
Then $\theta = \tan^{-1}x$
 $\therefore y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$
 $= \tan^{-1}(\tan 2\theta)$
 $= 2\theta$
 $= 2\tan^{-1}x$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}(2\tan^{-1}x)$
 $= 2\frac{d}{dx}(\tan^{-1}x)$
 $= 2 \times \frac{1}{1+x^2}$
 $= \frac{2}{1+x^2}$.

Exercise 1.2 | Q 9.03 | Page 30

Differentiate the following w.r.t. x : $\sin^{-1} \left(rac{1-x^2}{1+x^2}
ight)$





Let y = $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ Put $x = tan\theta$. Then $\theta = \tan^{-1}x$ $\therefore y = \sin^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right)$ $= \sin^{-1}(\cos 2\theta)$ $=\sin^{-1}\left[\sin\left(\frac{\pi}{2}-2\theta\right)\right]$ $=\frac{\pi}{2}-2\theta$ $=\frac{\pi}{2}-2\tan^{-1}x$ Differentiating w.r.t. x, we get $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{d}}{\mathrm{dx}} \left(\frac{\pi}{2} - 2 \tan^{-1} x \right)$ $=\frac{\mathrm{d}}{\mathrm{d}\mathbf{x}}\left(\frac{\pi}{2}\right)-2\frac{\mathrm{d}}{\mathrm{d}\mathbf{x}}\left(\tan^{-1}x\right)$ $= 0 - 2 \times \frac{1}{1 + r^2}$ $=\frac{-2}{1+x^2}.$

Exercise 1.2 | Q 9.04 | Page 30 Differentiate the following w.r.t. x : $\sin^{-1} \left(2x \sqrt{1-x^2} \right)$





Let
$$y = \sin^{-1} \left(2x\sqrt{1 - x^2} \right)$$

Put $x = \sin\theta$.
Then $\theta = \sin^{-1}x$
 $\therefore y = \sin^{-1} \left(2\sin\theta\sqrt{1 - \sin^2\theta} \right)$
 $= \sin^{-1}(2\sin\theta\cos\theta)$
 $= \sin^{-1}(\sin2\theta)$
 $= 2\theta$
 $= 2\sin^{-1}x$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} \left(2\sin^{-1}x \right)$
 $= 2 \frac{d}{dx} \left(\sin^{-1}x \right)$
 $= 2 \frac{d}{\sqrt{1 - x^2}}$
 $= \frac{2}{\sqrt{1 - x^2}}$
We can also put $x = \cos\theta$.
Then $\theta = \cos^{-1}x$
 $\therefore y = \sin^{-1} \left(2\cos\theta\sqrt{1 - \cos^2\theta} \right)$
 $= \sin^{-1}(2\cos\theta\sin\theta)$
 $= \sin^{-1}(\sin2\theta)$
 $= 2\theta$
 $= 2\cos^{-1}x$
Differentiating w.r.t. x, we get

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$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x} \left(2\cos^{-1} x \right)$$
$$= 2\frac{\mathrm{d}}{\mathrm{d}x} \left(\cos^{-1} x \right)$$
$$= 2 \times \frac{-1}{\sqrt{1 - x^2}}$$
$$= \frac{-2}{\sqrt{1 - x^2}}$$
Hence, $\frac{\mathrm{d}y}{\mathrm{d}x} = \pm \frac{2}{\sqrt{1 - x^2}}$.

Exercise 1.2 | Q 9.05 | Page 30 Differentiate the following w.r.t. $x : \cos^{-1}(3x - 4x^3)$

SOLUTION

Let $y = \cos^{-1}(3x - 4x^3)$ Put $x = \sin\theta$. Then $\theta = \sin^{-1}x$ $\therefore y = \cos^{-1}(3\sin\theta - 4\sin^3\theta)$ $= \cos^{-1}(\sin^2\theta)$ $= \cos^{-1}\left[\cos\left(\frac{\pi}{2} - 3\theta\right)\right]$ $= \frac{\pi}{2} - 3\theta$ $= \frac{\pi}{2} - 3\sin^{-1}x$ Differentiating w.r.t. x, we get $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2} - 3\sin^{-1}x\right)$ $= \frac{dy}{dx}\left(\frac{\pi}{2}\right) - 3\frac{d}{dx}(\sin^{-1}x)$





$$= 0 - 3 \times \frac{1}{\sqrt{1 - x^2}}$$
$$= \frac{-3}{\sqrt{1 - x^2}}.$$

Exercise 1.2 | Q 9.06 | Page 30

Differentiate the following w.r.t. x : $\cos^{-1}\left(rac{e^x-\ e^{-x}}{e^x+\ e^{-x}}
ight)$

SOLUTION

Let
$$y = \cos^{-1} \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right)$$

= $\cos^{-1} \left[\frac{e^x - \frac{1}{e^x}}{e^x + \frac{1}{e^x}} \right]$
= $\cos^{-1} \left(\frac{e^{2x} - 1}{e^{2x} + 1} \right)$
Put $e^x = \tan\theta$.

Then
$$\theta = \tan^{-1}(e^{X})$$

 $\therefore y = \cos^{-1}\left(\frac{\tan^{2}\theta - 1}{\tan^{2}\theta + 1}\right)$
 $= \cos^{-1}\left[-\left(\frac{1 - \tan^{2}\theta}{1 + \tan^{2}\theta}\right)\right]$
 $= \cos^{-1}(-\cos 2\theta)$
 $= \cos^{-1}[\cos(\pi - 2\theta)]$
 $= \pi - 2\theta$
 $= \pi - 2\tan^{-1}(e^{X})$
Differentiating w.r.t. x, we get

$$\begin{aligned} \frac{\mathrm{dy}}{\mathrm{dx}} &= \frac{\mathrm{d}}{\mathrm{dx}} \left[\pi - 2 \tan^{-1}(e^x) \right] \\ &= \frac{\mathrm{d}}{\mathrm{dx}} (\pi) - 2 \frac{\mathrm{d}}{\mathrm{dx}} \left[\tan^{-1}(e^x) \right] \\ &= 0 - 2 \times \frac{1}{1 + (e^x)^2} \cdot \frac{\mathrm{d}}{\mathrm{dx}} (e^x) \\ &= \frac{-2}{1 + e^{2x}} \times e^x \\ &= \frac{2e^x}{1 + e^{2x}}. \end{aligned}$$

Exercise 1.2 | Q 9.07 | Page 30

Differentiate the following w.r.t. x : $\cos^{-1} \ {(1-9^x) \over (1+9^x)}$

SOLUTION

Let
$$y = \cos^{-1} \frac{(1-9^x)}{(1+9^x)}$$

 $= \cos^{-1} \left[\frac{1-(3^x)^2}{1+(3^x)^2} \right]$
Put $3x = \tan\theta$.
Then $\theta = \tan^{-1}(3^x)$
 $\therefore y = \cos^{-1} \left(\frac{1-\tan^2\theta}{1+\tan^2\theta} \right)$
 $= \cos^{-1}(\cos 2\theta)$
 $= 2\theta$
 $= 2\tan^{-1}(3^x)$
Differentiating w.r.t. x, we get



$$\begin{aligned} \frac{\mathrm{dy}}{\mathrm{dx}} &= \frac{\mathrm{d}}{\mathrm{dx}} \left[2 \tan^{-1}(3^x) \right] \\ &= 2 \frac{\mathrm{d}}{\mathrm{dx}} \left[\tan^{-1}(3^x) \right] \\ &= 2 \times \frac{1}{1 + (3^x)^2} \cdot \frac{\mathrm{d}}{\mathrm{dx}} (3^x) \\ &= \frac{2}{1 + 3^{2x}} \times 3^x \log 3 \\ &= \frac{2 \cdot 3^x \log 3}{1 + 3^{2x}} \cdot \end{aligned}$$

Exercise 1.2 | Q 9.08 | Page 30

Differentiate the following w.r.t. x : $\sin^{-1}\left(rac{4^{x+rac{1}{2}}}{1-2^{4x}}
ight)$

SOLUTION

Let
$$y = \sin^{-1}\left(\frac{4^{x+\frac{1}{2}}}{1-2^{4x}}\right)$$

$$= \sin^{-1}\left[\frac{4^{x} \cdot 4^{\frac{1}{2}}}{1+(2^{2})^{2c}}\right]$$

$$= \sin^{-1}\left(\frac{2 \cdot 4^{x}}{1+4^{2x}}\right)$$
Put $4^{x} = \tan\theta$,
Then $\theta = \tan^{-1}(4^{x})$
 $\therefore y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan\theta}\right)$
 $= \sin^{-1}(\sin 2\theta)$
 $= 2\theta$

CLICK HERE



=
$$2\tan^{-1}(4^{x})$$

Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} [2\tan^{-1}(4^{x})]$
= $2\frac{d}{dx} [\tan^{-1}(4^{x})]$
= $2 \times \frac{1}{1+(4^{x})^{2}} \cdot \frac{d}{dx}(4^{x})$
= $\frac{2}{1+4^{2x}} \times 4^{x} \log 4$
= $\frac{2.4^{x} \log 4}{1+4^{2x}}$

Note: The answer can also be written as :

$$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{4^{\frac{1}{2}} \cdot 4^x \log 4}{1 + 4^{2x}}$$
$$= \frac{4^{x + \frac{1}{2}} \cdot \log 4}{1 + 4^{2x}}.$$

Exercise 1.2 | Q 9.09 | Page 30

Differentiate the following w.r.t. x : $\sin^{-1} \left(rac{1-25x^2}{1+25x^2}
ight)$





Let
$$y = \sin^{-1} \left(\frac{1-25x^2}{1+25x^2} \right)$$

$$= \sin^{-1} \left[\frac{1-(5x)^2}{1+(5x)^2} \right]$$
Put $5x = \tan\theta$.
Then $\theta = \tan^{-1}(5x)$
 $\therefore y = \sin^{-1} \left(\frac{1-\tan^2\theta}{1+\tan^2\theta} \right)$
 $= \sin^{-1}(\cos 2\theta)$
 $= \sin^{-1} \left[\sin\left(\frac{\pi}{2} - 2\theta\right) \right]$
 $= \frac{\pi}{2} - 2\theta$
 $= \frac{\pi}{2} - 2\tan^{-1}(5x)$
Differentiating w.r.t. x, we get
 $\therefore \frac{dy}{dx} = \frac{d}{dx} \left[\frac{\pi}{2} - 2\tan^{-1}(5x) \right]$
 $= \frac{d}{dx} \left(\frac{\pi}{2} \right) - 2\frac{d}{dx} \left[\tan^{-1}(5x) \right]$
 $= 0 - 2 \times \frac{1}{1+(5)^2} \cdot \frac{d}{dx} (5x)$
 $= \frac{-2}{1+25x^2} \times 5$
 $= \frac{-10}{1+25x^2}$.





Exercise 1.2 | Q 9.1 | Page 30

Differentiate the following w.r.t. x : $\sin^{-1} \, \left(rac{1-x^3}{1+x^3}
ight)$

SOLUTION

Let
$$y = \sin^{-1} \left(\frac{1-x^3}{1+x^3} \right)$$

$$= \sin^{-1} \left[\frac{1-\left(\frac{x^3}{2}\right)^2}{1+\left(\frac{x^3}{2}\right)^2} \right]$$
Put $x^{\frac{3}{2}} = \tan \theta$.
Then $\theta = \tan^{-1} \left(x^{\frac{3}{2}}\right)$
 $\therefore y = \sin^{-1} \left(\frac{1-\tan^2 \theta}{1+\tan^2 \theta}\right)$
 $= \sin^{-1}(\cos 2\theta)$
 $= \left[\sin\left(\frac{\pi}{2}-2\theta\right) \right]$
 $= \frac{\pi}{2} - 2\theta$
 $= \frac{\pi}{2} - 2 \tan^{-1} \left(x^{\frac{3}{2}}\right)$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} \left[\frac{\pi}{2} - 2 \tan^{-1} \left(x^{\frac{3}{2}}\right) \right]$
 $= \frac{d}{dx} \left(\frac{\pi}{2}\right) - 2 \frac{d}{dx} \left[\tan^{-1} \left(x^{\frac{3}{2}}\right) \right]$
 $= 0 - 2 \times \frac{1}{1+\left(x^{\frac{3}{2}}\right)^2} \cdot \frac{d}{dx} \left(x^{\frac{3}{2}}\right)$





$$= \frac{2}{1+x^3} \times \frac{3}{2}x^{\frac{1}{2}}$$
$$= -\frac{3\sqrt{x}}{1+x^3}.$$

Exercise 1.2 | Q 9.11 | Page 30

Differentiate the following w.r.t. x : $an^{-1}\left(rac{2x^{rac{5}{2}}}{1-x^5}
ight)$

SOLUTION

Let y = $\tan^{-1}\left(\frac{2x^{\frac{5}{2}}}{1-x^5}\right)$ Put $x^{\frac{5}{2}} = \tan \theta$. Then $\theta = \tan^{-1}\left(x^{\frac{5}{2}}\right)$ $\therefore y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$ $= \tan^{-1}(\tan 2\theta)$ $= 2\theta$ $= 2 \tan^{-1} \left(x^{\frac{5}{2}} \right)$ Differentiating w.r.t. x, we get $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{d}}{\mathrm{dx}} \left[2 \tan^{-1} \left(x^{\frac{5}{2}} \right) \right]$ $=2\frac{\mathrm{d}}{\mathrm{d}\mathbf{x}}\left[\mathrm{tan}^{-1}\left(x^{\frac{5}{2}}\right)\right]$ $= 2 \times \frac{1}{1 + \left(x^{\frac{5}{2}}\right)^2} \cdot \frac{\mathrm{d}}{\mathrm{d}x} \left(x^{\frac{5}{2}}\right)$ $=\frac{2}{1+x^5}\times\frac{5}{2}x^{\frac{3}{2}}$





$$=\frac{5x\sqrt{x}}{1+x^5}.$$

Exercise 1.2 | Q 9.12 | Page 30

Differentiate the following w.r.t. x : $\cot^{-1}\left(rac{1-\sqrt{x}}{1+\sqrt{x}}
ight)$

SOLUTION

Let
$$y = \cot^{-1}\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)$$

$$= \tan^{-1}\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right) \dots \left[\because \cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)\right]$$

$$= \tan^{-1}\left(\frac{1+\sqrt{x}}{1-1\times\sqrt{x}}\right)$$

$$= \tan^{-1}(1) + \tan^{-1}(\sqrt{x}) \dots \left[\because \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}x + \tan^{-1}y\right]$$

$$= \frac{\pi}{4} + \tan^{-1}(\sqrt{x})$$
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\pi}{4} + \tan^{-1}(\sqrt{x})\right]$$

$$= \frac{d}{dx}\left(\frac{\pi}{4}\right) + \frac{d}{dx}\left[\tan^{-1}(\sqrt{x})\right]$$

$$= 0 + \frac{1}{1+(\sqrt{x})^2} \cdot \frac{d}{dx}(\sqrt{x})$$

$$= \frac{1}{1+x} \times \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}(1+x)}.$$

Exercise 1.2 | Q 10.1 | Page 30

Differentiate the following w.r.t. x : $an^{-1} igg(rac{8x}{1-15x^2} igg)$

SOLUTION

Let
$$y = \tan^{-1}\left(\frac{8x}{1-15x^2}\right)$$

$$= \tan^{-1}\left[\frac{5x+3x}{1-(5x)(3x)}\right]$$

$$= \tan^{-1}(5x) + \tan^{-1}(3x)$$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}\left[\tan^{-1}(5x) + \tan^{-1}(3x)\right]$

$$= \frac{d}{dx}\left[\tan^{-1}(5x)\right] + \frac{d}{dx}\left[\tan^{-1}(3x)\right]$$

$$= \frac{1}{1+(5x)^2} \cdot \frac{d}{dx}(5x) + \frac{1}{1+(3x)^2} \cdot \frac{d}{dx}(3x)$$

$$= \frac{1}{1+25x^2} \times 5 + \frac{1}{1+9x^2} \times 3$$

$$= \frac{5}{1+25x^2} + \frac{3}{1+9x^2}.$$

Exercise 1.2 | Q 10.2 | Page 30

Differentiate the following w.r.t. x : $\cot^{-1}\left(rac{1+35x^2}{2x}
ight)$




Let
$$y = \cot^{-1}\left(\frac{1+35x^2}{2x}\right)$$

$$= \tan^{-1}\left(\frac{2x}{1+35x^2}\right) \dots \left[\because \cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)\right]$$

$$= \tan^{-1}\left[\frac{7x-5x}{1+(7x)(5x)}\right]$$

$$= \tan^{-1}(7x) - \tan^{-1}(5x)$$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\tan^{-1}(7x) - \tan^{-1}(5x)\right]$$

$$= \frac{d}{dx}\left[\tan^{-1}(x)\right] - \frac{d}{dx}\left[\tan^{-1}(5x)\right]$$

$$= \frac{1}{1+(7)^2} \cdot \frac{d}{dx}(7x) - \frac{1}{1+(5x)^2} \cdot \frac{d}{dx}(5x)$$

$$= \frac{1}{1+49x^2} \times 7 - \frac{1}{1+25x^2} \times 5$$

$$= \frac{7}{1+49x^2} - \frac{5}{1+25x^2}.$$

Exercise 1.2 | Q 10.3 | Page 30

Differentiate the following w.r.t. x : $an^{-1} \left(rac{2\sqrt{x}}{1+3x}
ight)$

SOLUTION

Let
$$y = \tan^{-1}\left(\frac{2\sqrt{x}}{1+3x}\right)$$

= $\tan^{-1}\left[\frac{3\sqrt{x}-\sqrt{x}}{1+(3\sqrt{x})(\sqrt{x})}\right]$

$$= \tan^{-1}(3\sqrt{x}) - \tan^{-1}(\sqrt{x})$$

Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{d}{dx} \left[\tan^{-1}(\sqrt{x}) - \tan^{-1}(\sqrt{x}) \right] \right]$$
$$= \frac{d}{dx} \left[\tan^{-(3\sqrt{x})} - \frac{d}{dx} \left[\tan^{-1}(\sqrt{x}) \right] \right]$$
$$= \frac{1}{1 + (3\sqrt{x})^2} \cdot \frac{d}{dx} (3\sqrt{x}) - \frac{1}{1 + (\sqrt{x})^2} \cdot \frac{d}{dx} (\sqrt{x})$$
$$= \frac{1}{1 + 9x} \times 3 \times \frac{1}{2\sqrt{x}} - \frac{1}{1 + x} \times \frac{1}{2\sqrt{x}}$$
$$= \frac{1}{2\sqrt{x}} \left[\frac{3}{1 + 9x} - \frac{1}{1 + x} \right].$$

Exercise 1.2 | Q 10.4 | Page 30

Differentiate the following w.r.t. x : $an^{-1} igg[rac{2^x+2}{1-3(4^x)} igg]$

SOLUTION

Let
$$y = \tan^{-1} \left[\frac{2^x + 2}{1 - 3(4^x)} \right]$$

= $\tan^{-1} \left[\frac{2^2 \cdot 2^x}{1 - 3(4^x)} \right]$
= $\tan^{-1} \left[\frac{4 \cdot 2^x}{1 - 3(4^x)} \right]$
= $\tan^{-1} \left[\frac{3 \cdot 2^x + 2^x}{1 - (3 \cdot 2^x \times 2^x)} \right]$
= $\tan^{-1} (3 \cdot 2^x) + \tan^{-1} (2^x)$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx} \left[\tan^{-1}(3.2^{x}) + \tan^{-1}(2^{x}) \right]$$

$$= \frac{d}{dx} \left[\tan^{-1}(3.3x^{x}) \right] + \frac{d}{dx} \left[\tan^{-1}(2^{x}) \right]$$

$$= \frac{1}{1 + (3.2^{x})^{2}} \cdot \frac{d}{dx} (3.2^{x}) + \frac{1}{1 + (2^{x})^{2}} \cdot \frac{d}{dx} (2^{x})$$

$$= \frac{1}{1 + 9(2^{2x})} \times 3 \times 2^{x} \log 2 + \frac{1}{1 + 2^{2x}} \times 2^{x} \log 2$$

$$= 2^{x} \log 2 \left[\frac{3}{1 + 9(2^{2x})} + \frac{1}{1 + 2^{2x}} \right].$$

Exercise 1.2 | Q 10.5 | Page 30

Differentiate the following w.r.t. x : $an^{-1} igg(rac{2^x}{1+2^{2x+1}} igg)$

SOLUTION

Let
$$y = \tan^{-1} \left(\frac{2^x}{1 + 2^{2x+1}} \right)$$

 $= \tan^{-1} \left[\frac{2 \cdot 2^x - 2^x}{1 + (2 \cdot 2^x)(2^x)} \right]$
 $= \tan^{-1}(2 \cdot 2^x) - \tan^{-1}(2^x)$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx} \left[\tan^{-1}(2 \cdot 2^x) - \tan^{-1}(2^x) \right]$
 $= \frac{d}{dx} \left[\tan^{-1}(2 \cdot 2^x) \right] - \frac{d}{dx} \left[\tan^{-1}(2^x) \right]$
 $= \frac{1}{1 + (2 \cdot 2^x)^2} \cdot \frac{d}{dx} (2 \cdot 2^x) - \frac{1}{1 + (2^x)^2} \cdot \frac{d}{dx} (2^x)$



$$= \frac{1}{1+4(2^{2x})} \times 2 \times 2^x \log 2 - \frac{1}{1+2^{2x}} \times 2^x \log 2$$
$$= 2^x \log 2 \left[\frac{2}{1+4(2^{2x})} - \frac{1}{1+2^{2x}} \right].$$

Exercise 1.2 | Q 10.6 | Page 30

Differentiate the following w.r.t. x : $\cot^{-1} \left(rac{a^2 - 6x^2}{5ax}
ight)$

SOLUTION

Let
$$y = \cot^{-1}\left(\frac{a^2 - 6x^2}{5ax}\right)$$

$$= ta^{-1}\left(\frac{5ax}{a^2 - 6x^2}\right) \dots \left[\because \cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)\right]$$

$$= \tan^{-1}\left[\frac{5\left(\frac{x}{a}\right)}{1 - 6\left(\frac{x}{a}\right)^2}\right] \dots \left[\text{Dividing by a}^2\right]$$

$$= \tan\left[\frac{3\left(\frac{x}{a}\right) + 2\left(\frac{x}{a}\right)}{1 - 3\left(\frac{x}{a}\right) \times 2\left(\frac{x}{a}\right)}\right]$$

$$= \tan^{-1}\left(\frac{3x}{a}\right) + \tan^{-1}\left(\frac{2x}{a}\right)$$
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{d}{dx}\left[\tan^{-1}\left(\frac{3x}{a}\right) + \tan^{-1}\left(\frac{x}{a}\right)\right]$$

$$= \frac{1}{dx}\left[\tan^{-1}\left(\frac{3x}{a}\right)\right] + \frac{d}{dx}\left[\tan^{-1}\left(\frac{2x}{a}\right)\right]$$

$$= \frac{1}{1 + \left(\frac{9x^2}{a^2}\right)} \times \frac{3}{a} \times 1 + \frac{1}{1 + \left(\frac{4x^2}{a^2}\right)} \times \frac{2}{a} \times 1$$

$$= \frac{a^2}{a^2 + 9x^2} \times \frac{3}{a} + \frac{a^2}{a^2 + 4x^2} \times \frac{2}{a}$$
$$= \frac{3a}{a^2 + 9x^2} + \frac{2a}{a^2 + 4x^2}.$$

Exercise 1.2 | Q 10.7 | Page 30

Differentiate the following w.r.t. x : $\tan^{-1} \left(\frac{a + b \tan x}{b - a \tan x} \right)$

SOLUTION

Let
$$y = \tan^{-1}\left(\frac{a+b\tan x}{b-a\tan x}\right)$$

$$= \tan^{-1}\left[\frac{\frac{a}{b}+\tan x}{1-\frac{a}{b}\cdot\tan x}\right]$$

$$= \tan^{-1}\left(\frac{a}{b}\right) + \tan^{-1}(\tan x)$$

$$= \tan^{-1}\left(\frac{a}{b}\right) + x$$
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{d}{dx}\left[\tan^{-1}\left(\frac{a}{b}\right) + x\right]$$

$$= \frac{d}{dx}\left[\tan^{-1}\left(\frac{a}{b}\right)\right] + \frac{d}{dx}(x)$$

$$= 0 + 1$$

$$= 1.$$

Exercise 1.2 | Q 10.8 | Page 30

Differentiate the following w.r.t. x : $an^{-1}igg(rac{5-x}{6x^2-5x-3}igg)$





Let
$$y = \tan^{-1}\left(\frac{5-x}{6x^2-5x-3}\right)$$

$$= \tan^{-1}\left[\frac{5-x}{1+(6x^2-5x-4)}\right]$$

$$= \tan^{-1}\left[\frac{(2x-1)-(3x-4)}{1+(2x+1)(3x-4)}\right]$$

$$= \tan^{-1}(2x+1) - \tan^{-1}(3x-4)$$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}\left[\tan^{-1}(2x+1)\right] - \frac{d}{dx}\left[\tan^{-1}(3x-4)\right]$

$$= \frac{1}{1+(2x+1)^2} \cdot \frac{d}{dx}(2x+1) - \frac{1}{1+(3x-4)^2} \cdot \frac{d}{dx}(3x-4)$$

$$= \frac{1}{1+(2x+1)^2} \cdot (2 \times 1+0) - \frac{1}{1+(3x-4)^2} \cdot (3 \times 1-0)$$

$$= \frac{2}{1+(2x+1)^2} - \frac{3}{1+(3x-4)^2}.$$

Exercise 1.2 | Q 10.9 | Page 30

Differentiate the following w.r.t. x : $\cot^{-1} \left(rac{4-x-2x^2}{3x+2}
ight)$





Let
$$y = \cot^{-1}\left(\frac{4-x-2x^2}{3x+2}\right)$$

= $\tan^{-1}\left(\frac{3x+2}{4-x-2x^2}\right) \dots \left[\because \cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)\right]$
= $\tan^{-1}\left[\frac{3x+2}{1-(2x^2+x-3)}\right]$
= $\tan^{-1}\left[\frac{(2x+3)+(x-1)}{1-(2x+3)(x-1)}\right]$
= $\tan^{-1}(2x+3) + \tan^{-1}(x-1)$
Differentiating w.r.t. x, we get
 $\frac{dy}{dx} = \frac{d}{dx}\left[\tan^{-1}(2x+3) + \tan^{-1}(x-1)\right]$
= $\frac{1}{4x}\left[\tan^{-1}(2x+3)\right] + \frac{d}{dx}\left[\tan^{-1}(x-1)\right]$
= $\frac{1}{1+(2x+3)^2} \cdot \frac{d}{dx}(2x+3) + \frac{1}{1+(x-1)^2} \cdot \frac{d}{dx}(x-1)$
= $\frac{1}{1+(2x+3)^2} \cdot (2 \times 1+0) + \frac{1}{1+(x-1)^2} \cdot (1-0)$
= $\frac{2}{1+(2x+3)^2} + \frac{1}{1+(x-1)^2}$.
EXERCISE 1.3 [PAGES 39 - 40]
Exercise 1.3 [Q.1.1] Page 39

Differentiate the following w.r.t. x : $rac{{{\left({x + 1}
ight)}^2 }}{{{\left({x + 2}
ight)}^3 {\left({x + 3}
ight)}^4 }}$

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Let
$$y = \frac{(x+1)^2}{(x+2)^3(x+3)^4}$$

Then, $\log y = \log \left[\frac{(x+1)^2}{(x+2)^3(x+3)^4} \right]$
 $= \log(x+1)^2 - \log(x+2)^3 - \log(x+3)^4$
 $= 2\log(x+1) - 3\log(x+2) - 4\log(x+3)$
Differentiating w.r.t. x, we get
 $\frac{1}{y} \frac{dy}{dx} = 2 \frac{d}{dx} [\log(x+1)] - 3 \frac{d}{dx} [\log(x+2)] - 4 \frac{d}{dx} [\log(x+3)]$
 $= 2 \times \frac{1}{x+1} \cdot \frac{d}{dx} (x+1) - 3 \times \frac{1}{x+2} \cdot \frac{d}{dx} (x+2) - 4 \times \frac{1}{x+3} \cdot \frac{d}{dx} (x+3)$
 $= \frac{2}{x+1} \cdot (1+0) - \frac{3}{x+2} \cdot (1+0) - \frac{4}{x+3} \cdot (1+0)$
 $\therefore \frac{dy}{dx} = y \left[\frac{2}{x+1} - \frac{3}{x+2} - \frac{4}{x+3} \right]$
 $= \frac{(x+1)^2}{(x+2)^2(x+3)^4} \cdot \left[\frac{2}{x+1} - \frac{3}{x+2} - \frac{4}{x+3} \right].$

Exercise 1.3 | Q 1.2 | Page 39

Differentiate the following w.r.t. x : $\sqrt[3]{\frac{4x-1}{(2x+3)(5-2x)^2}}$





Let
$$y = \sqrt[3]{\frac{4x-1}{(2x+3)(5-2x)^2}}$$

Then $\log y = \log \left[\frac{4x-1}{(2x+3)(5-2x)^2}\right]^{\frac{1}{3}}$
 $= \frac{1}{3}\log \left[\frac{4x-1}{(2x+3)(5-2x)^2}\right]^{\frac{1}{3}}$
 $= \frac{1}{3}\left[\log(4x-1) - \log(2x+3)(5-2x)^2\right]$
 $= \frac{1}{3}\log(4x-1) - \frac{1}{3}\log(2x+3) - \frac{2}{3}\log(5-2x)$
Differentiating both sides w.r.t. x, we get
 $\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{3}\frac{d}{dx}[\log(4x-1)] - \frac{1}{3}\frac{d}{dx}[\log(2x+3)] - \frac{2}{3}\frac{d}{dx}[\log(5-2x)]$
 $= \frac{1}{3} \times \frac{1}{4x-1} \cdot \frac{d}{dx}(4x-1) - \frac{1}{3} \times \frac{1}{2x+3} \cdot \frac{d}{dx}(2x+3) - \frac{2}{3} \times \frac{1}{5-2x} \cdot \frac{d}{dx}(5-2x)$
 $= \frac{1}{3(4x-1)} \cdot (4 \times 1 - 0) - \frac{1}{3(2x+3)} \cdot (2 \times 1 + 0) - \frac{2}{3(5-2x)} \cdot (0 - 2 \times 1)$
 $\therefore \frac{dy}{dx} = y \left[\frac{4}{3(4x-1)} - \frac{2}{3(2x+3)} + \frac{4}{3(5-2x)}\right]$
 $= \sqrt[3]{\frac{4x-1}{(2x+3)(5-2x)^2}} \left[\frac{4}{3(4x-1)} - \frac{2}{3(2x+3)} + \frac{4}{3(5-2x)}\right].$

Exercise 1.3 | Q 1.3 | Page 39

Differentiate the following w.r.t. x : $\left(x^2+3
ight)^{rac{3}{2}}.\sin^3 2x.2^{x^2}$





Let
$$y = (x^2 + 3)^{\frac{3}{2}} \cdot \sin^3 2x \cdot 2^{x^2}$$

Then $\log y = \log[x^2 + 3)^{\frac{3}{2}} \cdot \sin^3 2x \cdot 2^{x^2}$
 $= \log(x^2 + 3)^{\frac{3}{2}} + \log \sin^3 2x + \log 2^{x^2}$
 $= \frac{3}{2}\log(x^2 + 3) + 3\log(\sin 2x) + x^2 \cdot \log 2$
Differentiating both sides w.r.t. x, we get
 $\frac{1}{y} \cdot \frac{dy}{dx} = \frac{3}{2} \frac{d}{dx} [\log(x^2 + 3)] + 3 \frac{d}{dx} [\log(\sin 2x)] + \log 2 \cdot \frac{d}{dx} (x^2)$
 $= \frac{3}{2} \times \frac{1}{x^2 + 3} \cdot \frac{d}{dx} (x^2 + 3) + 3 \times \frac{1}{\sin 2x} \cdot \frac{d}{dx} (\sin 2x) + \log 2 \times 2x$
 $= \frac{3}{2(x^2 + 3)} \cdot (2x + 0) + \frac{3}{\sin 2x} \times \cos 2x \cdot \frac{d}{dx} (2x) + 2x \log 2$
 $= \frac{6x}{2(x^2 + 3)} + 3 \cot 2x \times 2 + 2x \log 2$
 $\therefore \frac{dy}{dx} = y [\frac{3x}{x^2 + 3} + 6 \cot 2x + 2x \log 2]$
 $= (x^2 + 3)^{\frac{3}{2}} \cdot \sin^3 2x \cdot 2^{x^2} [\frac{3x}{x^2 + 3} + 6 \cot 2x + 2x \log 2].$

Exercise 1.3 | Q 1.4 | Page 39

Differentiate the following w.r.t. x :
$$rac{\left(x^2+2x+2
ight)^{rac{3}{2}}}{\left(\sqrt{x}+3
ight)^3\left(\cos x
ight)^x}$$





Let
$$y = \frac{(x^2 + 2x + 2)^{\frac{3}{2}}}{(\sqrt{x} + 3)^3(\cos x)^x}$$

Then $\log y = \log \left[\frac{(x^3 + 2x + 2)^{\frac{3}{2}}}{(\sqrt{x} + 3)^3(\cos x)^x}\right]^{\frac{3}{2}}$
 $= \log(x^2 + 2x + 2)^{\frac{3}{2}} - \log(\sqrt{x} + 3)^3(\cos x)^x$
 $= \frac{3}{2}\log(x^2 + 2x + 2) - 3\log(\sqrt{x} + 3) - x\log(\cos x)$
Differentiating both sides w.r.t. x, we get
 $\frac{1}{y} \cdot \frac{dy}{dx} = \frac{3}{2} \frac{d}{dx} [\log(x^2 + 2x + 2)] - 3\frac{d}{dx} [\log(\sqrt{x} + 3)] - \frac{d}{dx} [x\log(\cos x)]$
 $= \frac{3}{2} \times \frac{1}{x^2 + 2x + 2} \cdot \frac{d}{dx} (x^2 + 2x + 2) - 3 \times \frac{1}{\sqrt{x} + 3} \cdot \frac{d}{dx} (\sqrt{x} + 3) - \left\{ x \frac{d}{dx} [\log(\cos x)] + \log(\cos x) \cdot \frac{d}{dx} (x) + \frac{3}{2(x^2 + 2x + 2)} \times (2x + 2 \times 1 + 0) - \frac{3}{\sqrt{x} + 3} \times \left(\frac{1}{2\sqrt{x} + 0}\right) - x \times \frac{1}{\cos x} \cdot \frac{d}{dx} (\cos x) + \log(\cos x) \times 1 \right\}$
 $= \frac{3(2x + 2)}{2(x^2 + 2x + 2)} - \frac{3}{2\sqrt{x}(\sqrt{x} + 3)} - \left\{ x \times \frac{1}{\cos x} \cdot (-\sin x) + \log(\cos x) \right\}$
 $\therefore \frac{dy}{dx} = y \left[\frac{3(x + 1)}{x^3 + 2x + 2} - \frac{3}{2\sqrt{x}(\sqrt{x} + 3)} + x \tan x - \log(\cos x) \right]$
 $= \frac{(x^2 + 2x + 2)^{\frac{3}{2}}}{(\sqrt{x} + 3)^3(\cos x)^x} \left[\frac{3(x + 1)}{x^2 + 2x + 2} - \frac{3}{2\sqrt{x}(\sqrt{x} + 3)} + x \tan x - \log(\cos x) \right].$

Exercise 1.3 | Q 1.5 | Page 39

Differentiate the following w.r.t. x : $rac{x^5 \cdot an^3 \, 4x}{\sin^2 3x}$





Let
$$y = \frac{x^5 \cdot \tan^3 4x}{\sin^2 3x}$$

Then $\log y = \log \left[\frac{x^5 \cdot \tan^3 4x}{\sin^{23} x} \right]$
 $= \log x^5 + \log \tan^3 ax - \log \sin^2 3x$
 $= 5\log x + 3\log (\tan 4x) - 2\log (\sin 3x)$
Differentiating both sides w.r.t. x, we get
 $\frac{1}{y} \cdot \frac{dy}{dx} = 5 \frac{d}{dx} (\log x) + 3 \frac{d}{dx} [\log(\tan 4x)] - 2 \frac{d}{dx} [\log(\sin 3x)]$
 $= 5 \times \frac{1}{x} + 3 \times \frac{1}{\tan 4x} \cdot \frac{d}{dx} (\tan 4x) - 2 \times \frac{1}{\sin 3x} \cdot \frac{d}{dx} (\sin 3x)$
 $= \frac{5}{x} + 3 \times \frac{1}{\tan 4x} \times \sec^{24} x \cdot \frac{d}{dx} (4x) - 2 \times \frac{1}{\sin 3x} \times \cos 3x \cdot \frac{d}{dx} (3x)$
 $= \frac{5}{x} + 3 \cdot \frac{\cos 4x}{\sin 4x} \times \frac{1}{\cos^{24} x} \times 4 - 2 \cot 3x \times 3$
 $= \frac{5}{x} + \frac{24}{2 \sin 4x \cdot \cos 4x} - 6 \cot 3x$
 $\therefore \frac{dy}{dx} = y \left[\frac{5}{x} + \frac{24}{\sin 8x} - 6 \cot 3x \right]$

Exercise 1.3 | Q 1.6 | Page 39

Differentiate the following w.r.t. $\mathsf{x}: x^{ an^{-1}x}$





Let
$$y = x^{\tan^{-1}x}$$

Then $\log y = \log(x^{\tan^{-1}x}) = (\tan^{-1}x)(\log x)$
Differentiating both sides w.r.t. x, we get
 $\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx} [(\tan^{-1}x)(\log x)]$
 $= (\tan^{-1}x) \cdot \frac{d}{dx}(\log x) + (\log x) \cdot \frac{d}{dx}(\tan^{-1}x)$
 $= (\tan^{-1}x) \times \frac{1}{x} + (\log x) \times \frac{1}{1+x^2}$
 $\therefore \frac{dy}{dx} = y \Big[\frac{\tan^{-1}x}{x} + \frac{\log x}{1+x^2} \Big]$
 $= x^{\tan^{-1}x} \Big[\frac{\tan^{-1}x}{x} + \frac{\log x}{1+x^2} \Big].$

Exercise 1.3 | Q 1.7 | Page 39

Differentiate the following w.r.t. $x : (sin x)^{x}$

SOLUTION

Let $y = (\sin x)^{x}$ Then $\log y = \log(\sin x)^{x} = x \cdot \log(\sin x)$ Differentiating both sides w.r.t. x, we get $\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx} [x \cdot \log(\sin x)]$ $= x \cdot \frac{d}{dx} [\log(\sin x)] + \log(\sin x) \cdot \frac{d}{dx} (x)$ $= x \times \frac{1}{\sin x} \cdot \frac{d}{dx} (\sin x) + \log(\sin x) \times 1$





$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = y \left[x \times \frac{1}{\sin x} \cdot \cos x + \log(\sin x) \right]$$
$$= (\sin x)^{X} [x \cot x + \log (\sin x)].$$

Exercise 1.3 | Q 1.8 | Page 39

Differentiate the following w.r.t. $x : (\sin x^{X})$

SOLUTION

Let
$$y = (\sin x^{\chi})$$

Then $\frac{dy}{dx} = \frac{d}{dx}[(\sin x^{x})]$
 $\therefore \frac{dy}{dx} = \cos(x^{x}) \cdot \frac{d}{dx}(x^{x})$...(1)
Let $u = x^{\chi}$
Then log $u = \log x^{\chi} = x \cdot \log x$
Differentiating both sides w.r.t. x , we get
 $\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx}(x \cdot \log x)$
 $= x \cdot \frac{d}{dx}(\log x) + (\log x) \cdot \frac{d}{dx}(x)$
 $= x \times \frac{1}{x} + (\log x) \times 1$
 $\therefore \frac{du}{dx} = u(1 + \log x)$
 $\therefore \frac{d}{dx}(x^{x}) = x^{x}(1 + \log x)$...(2)
From (1) and (2), we get
 $\frac{dy}{dx} = \cos(x^{x}) \cdot x^{x}(1 + \log x)$.



Exercise 1.3 | Q 2.1 | Page 40

Differentiate the following w.r.t. $x : x^e + x^x + e^x + e^e$

SOLUTION

Let
$$y = x^e + x^x + e^x + e^e$$

Let $u = x^x$
Then log $u = \log x^x = x \log x$
Differentiating both sides w.r.t. x , we get
 $\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx} (x \log x)$
 $= x \cdot \frac{d}{dx} (\log x) + (\log x) \cdot \frac{d}{dx} (x)$
 $= x \times \frac{1}{x} + (\log x)(1)$
 $\therefore \frac{du}{dx} = u(1 + \log x) = x^x (1 + \log x) \quad ...(1)$
Now, $y = x^e + u + e^x + e^e$
 $\therefore \frac{dy}{dx} = \frac{d}{dx} (x^e) + \frac{du}{dx} + \frac{d}{dx} (e^x) + \frac{d}{dx} (e^x)$
 $= ex^{e-1} + x^x (1 + \log x) + e^x + 0 \quad ...[By (1)]$
 $= ex^{e-1} + e^x + x^x (1 + \log x).$

Exercise 1.3 | Q 2.2 | Page 40 Differentiate the following w.r.t. x : $x^{x^x} + e^{x^x}$





Let $y = x^{x^x} + e^{x^x}$ Put u = x^{x^x} and $v = e^{x^x}$ Then y = u + v $\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}u}{\mathrm{d}x} + \frac{\mathrm{d}v}{\mathrm{d}x}$...(1) Take $u = x^{x^x}$ $\therefore \log u = \log x^{x^x} = x^x \cdot \log x$ Differentiating both sides w.r.t. x, we get $\frac{1}{u} \cdot \frac{\mathrm{du}}{\mathrm{dx}} = \frac{\mathrm{d}}{\mathrm{dx}} \left(x^2 \cdot \log x \right)$ $= x^{x} \cdot \frac{\mathrm{d}}{\mathrm{d}x} (\log x) + (\log x) \cdot \frac{\mathrm{d}}{\mathrm{d}x} (x^{x})$ $= x^x \times \frac{1}{x} + (\log x) \cdot \frac{\mathrm{d}}{\mathrm{d}x} (x^3)$...(2) To find $\frac{d}{dx}(x^3)$ Let $\omega = x^{x}$. Then $\log \omega = x \log x$ Differentiating both sides w.r.t. x, we get $\frac{1}{\omega} \cdot \frac{\mathrm{d}\omega}{\mathrm{d}\mathbf{x}} = \frac{\mathrm{d}}{\mathrm{d}\mathbf{x}} (x\log x)$ $= x. \frac{\mathrm{d}}{\mathrm{d}x}(\log x) + (\log x). \frac{\mathrm{d}}{\mathrm{d}x}(x)$ $=x imesrac{1}{x}+(\log x) imes 1$ $\therefore \frac{\mathrm{d}\omega}{\mathrm{d}\mathbf{x}} = \omega(1 + \log x)$ $\therefore \frac{\mathrm{d}}{\mathrm{d}x}(x^x) = x^x(1 + \log x)$...(3) : from (2),



$$\frac{1}{u} \cdot \frac{\mathrm{du}}{\mathrm{dx}} = x^{x} \times \frac{1}{x} + (\log x) \cdot x^{x} (1 + \log x)$$

$$\therefore \frac{\mathrm{du}}{\mathrm{dx}} = y \left[x^{x} \times \frac{1}{x} + (\log x) \cdot x^{x} (1 + \log x) \right]$$

$$= x^{x^{x}} \cdot x^{x} \left[\frac{1}{x} + (\log x) \cdot (1 + \log x) \right]$$

$$= x^{x^{x}} \cdot x^{x} \cdot \log x \left[1 + \log x + \frac{1}{x \log x} \right] \quad \dots (4)$$

Also, $v = e^{x^{x}}$
$$\therefore \frac{\mathrm{dv}}{\mathrm{dx}} = \frac{\mathrm{d}}{\mathrm{dx}} (e^{x^{x}})$$

$$= e^{x^{x}} \cdot \frac{\mathrm{d}}{\mathrm{dx}} (e^{x^{x}})$$

$$= e^{x^{x}} \cdot x^{x} (1 + \log x) \qquad \dots (5)$$

From (1),(4) and (5), we get \qquad \dots (By (3)]
$$\frac{\mathrm{dy}}{\mathrm{dx}} = x^{x^{x}} \cdot x^{x} \cdot \log x \left[1 + \log x + \frac{1}{x \log x} \right] + e^{x^{x}} \cdot x^{x} (1 + \log x).$$

Exercise 1.3 | Q 2.3 | Page 40

Differentiate the following w.r.t. $x : (logx)^{x} - (cos x)^{cotx}$

SOLUTION

Let
$$y = (\log x)^{x} - (\cos x)^{\cot x}$$

Put $u = (\log x)^{x}$ and $v = (\cos x)^{\cot x}$
Then $y = u - v$
 $\therefore \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$...(1)
Take $u = (\log x)^{x}$
 $\therefore \log u = \log(\log x)^{x} = x.\log(\log x)$
Differentiating both sides w.r.t. x, we get

$$\frac{1}{u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x} [x \cdot \log(\log x)]$$

$$= x \frac{\mathrm{d}}{\mathrm{d}x} [\log(\log x)] + \log(\log x) \cdot \frac{\mathrm{d}}{\mathrm{d}x}(x)$$

$$= x \times \frac{1}{\log x} \cdot \frac{\mathrm{d}}{\mathrm{d}x} (\log x) + \log(\log x) \times 1$$

$$= x \times \frac{1}{\log x} \times \frac{1}{x} + \log(\log x)$$

$$\therefore \frac{\mathrm{d}u}{\iota} dx = u \left[\frac{1}{\log x} + \log(\log x) \right]$$

$$= (\log x) x \left[\frac{1}{\log x} + \log(\log x) \right] \qquad \dots (2)$$

Also v = $(\cos x)^{\cot x}$

$$\therefore \log c = \log(\cos x)^{\cot x} = (\cot x).(\log \cos x)$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{v} = \frac{dv}{dx} = \frac{d}{dx}[(\cot x).\log(\cos x)]$$

$$= (\cot x).\frac{d}{dx}(\log \cos x) + (\log \cos x).\frac{d}{dx}(\cot x)$$

$$= \cot x \times \frac{1}{\cos x} \cdot \frac{d}{dx}(\cos x) + (\log \cos x)(-\csc^2 x)$$

$$= \cot x \times \frac{1}{\cos x} \times (-\sin x) - (\csc^2 x)(\log \cos x)$$

$$\therefore \frac{dv}{dx} = v \left[\frac{1}{\tan x} \times (-\tan x) - (\csc^2 x)(\log \cos x)\right]$$

$$= -(\cos x)^{\cot x} [1 + (\csc^2 x)(\log \cos x)]$$

From (1), (2) and (3), we get

$$\therefore \frac{dv}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x)\right] + (\cos x)^{\cot x} [1 + (\csc^2 x)(\log \cos x)].$$

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Exercise 1.3 | Q 2.4 | Page 40

Differentiate the following w.r.t. x : $x^{x^x} + (\log x)^{\sin x}$

SOLUTION

Let
$$y = x^{x^{x}} + (\log x)^{\sin x}$$

Put $u = x^{e^{x}}$ and $v = (\log x)^{\sin x}$
Then $y = u + v$
 $\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$...(1)
Take $u = x^{e^{x}}$
 $\therefore \log u = \log x^{e^{x}} = e^{x} \cdot \log x$
Differentiating both sides w.r.t. x, we get
 $\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx}(e^{x}\log x)$
 $= e^{x} \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(e^{x})$
 $= e^{x} \cdot \frac{1}{x} + (\log x)(e^{x})$
 $\therefore \frac{du}{dx} = y \left[\frac{e^{x}}{x} + e^{x} \cdot \log x \right]$
 $= e^{x} \cdot x^{e^{x}} \left[\frac{1}{x} + \log x \right]$...(2)
Also, $v = (\log x)^{\sin x}$
 $\therefore \log v = \log(\log x)^{\sin x} = (\sin x).(\log \log x)$
Differentiating both sides w.r.t. x, we get
 $\frac{1}{v} \cdot \frac{dv}{dx} = \frac{d}{dx} [(\sin x) \cdot (\log \log x)]$

$$= (\sin x) \cdot \frac{\mathrm{d}}{\mathrm{d}x} \left[(\log \log x) + (\log \log x) \cdot \frac{\mathrm{d}}{\mathrm{d}x} (\sin x) \right]$$



$$= \sin x \times \frac{1}{\log x} \cdot \frac{d}{dx} (\log x) + (\log \log x) \cdot (\cos x)$$

$$\therefore \frac{dv}{dx} = v \left[\frac{\sin x}{\log x} \times \frac{1}{x} + (\cos x) (\log \log x) \right]$$

$$= (\log x)^{\sin x} \left[\frac{\sin x}{x} \log x + (\cos x) (\log \log x) \right] \quad ...(2)$$

From (1), (2) and (3), we get
$$\frac{dy}{dx} - e^x \cdot x^{e^x} \left[\frac{1}{x} + \log x \right] + (\log x)^{\sin x} \left[\frac{\sin x}{x \log x} + (\cos x) (\log \log x) \right].$$

Exercise 1.3 | Q 2.5 | Page 40

Differentiate the following w.r.t. $x : e^{tanx} + (logx)^{tanx}$

SOLUTION

Let
$$y = e^{\tan x} + (\log x)^{\tan x}$$

Put $u = (\log x)^{\tan x}$
 $\therefore \log u = \log (\log x)^{\tan x} = (\tan x).(\log \log x)$
Differentiating both sides w.r.t. x, we get
 $\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx} [(\tan x). (\log \log x)]$
 $= (\tan x) \cdot \frac{d}{dx} (\log \log x) + (\log \log x) \cdot \frac{d}{dx} (\tan x)$
 $= \tan x \times \frac{1}{\log x} \cdot \frac{d}{dx} (\log x) + (\log \log x) (\sec^2 x)$
 $= \tan x \times \frac{1}{\log x} \times \frac{1}{x} + (\log \log x) (\sec^2 x)$
 $\therefore du'/dx = u \left[\frac{\tan x}{x \log x} + (\log \log x) (\sec^2 x) \right]$

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$$= (\log x)^{\tan x} \left[\tan x (x \log x) + (\log \log x) (\sec^2 x) \right]$$

Now, $y = e^{\tan x} + u$
 $\therefore \frac{dy}{dx} = \frac{d}{dx} (e^{\tan x}) + \frac{du}{dx}$
 $= e^{\tan x} \cdot \frac{d}{dx} (\tan x) + \frac{du}{dx}$
 $= e^{\tan x} \cdot \sec^2 x + (\log x)^{\tan x} \left[\frac{\tan x}{x \log x} + (\log \log x) (\sec^2 x) \right].$

Exercise 1.3 | Q 2.6 | Page 40

Differentiate the following w.r.t. $x : (\sin x)^{tanx} + (\cos x)^{cotx}$

SOLUTION

Let
$$y = (\sin x)^{\tan x} + (\cos x)^{\cot x}$$

Put $u = (\sin x)^{\tan x}$ and $v = (\cos x)^{\cot x}$
Then $y = u + v$
 $\therefore \frac{dy}{dx} = \frac{dy}{dx} + \frac{dv}{dx}$...(1)
Take $u = (\sin x)^{\tan x}$
 $\therefore \log u = \log(\sin x)^{\tan x} = (\tan x).(\log \sin x)$
Differentiating both sides w.r.t. x, we get
 $\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx}[(\tan x)(\log \sin x)]$
 $= (\tan x) \cdot \frac{d}{dx}(\log \sin x) + (\log \sin x) \cdot \frac{d}{dx}(\tan x)$
 $= \frac{\tan x}{\sin x} \cdot \frac{d}{dx}(\sin x) + (\log \sin x)(\sec^2 x)$





$$= \frac{\sin x}{\sin x} \cdot \cos x + (\sec^2 x)(\log \sin x)$$

$$= 1 + (\sec^2 x)(\log \sin x)$$

$$\therefore \frac{du}{dx} = y [1 + (\sec^2 x)(\log \sin x)]$$

$$= (\sin x)^{\tan x} [1 + (\sec^2 x)(\log \sin x)] \quad ...(2)$$
Also, $v = (\cos x)^{\cot x}$

$$\therefore \log v = \log(\cos x)^{\cot x} = (\cot x).(\log \cos x)$$
Differentiating both sides w.r.t. x, we get
$$\frac{1}{v} \cdot \frac{dv}{dx} = \frac{d}{dx} [(\cot x) \cdot (\log \cos x)]$$

$$= (\cot x) \cdot \frac{d}{dx} (\log \cos x) + (\log \cos x) \cdot \frac{d}{dx} (\cot x)$$

$$= \cot x \times \frac{1}{\cos x} \cdot \frac{d}{dx} (\cos x) + (\log \cos x) \cdot (-\csc^2 x)$$

$$= \cot x \times \frac{1}{\cos x} \times (-\sin x) - (\csc^2 x)(\log \cos x)$$

$$\therefore \frac{dv}{dx} = v [\frac{1}{\tan x} \times (-\tan x) - (\csc^2 x)(\log \cos x)]$$

$$= -(\cos x)^{\cot x} [1 + (\csc^2 x)(\log \cos x)] \quad ...(3)$$
From (1), (2) and (3), we get
$$\frac{dy}{dx} = (\sin x)^{\tan x} [1 + (\sec^2 x)(\log \sin x)] - (\cos x)^{\cot x} [1 + (\csc^2 x)(\log \cos x)].$$

Exercise 1.3 | Q 2.7 | Page 40 Differentiate the following w.r.t. x : $10^{x^x} + x^{x(10)} + x^{10x}$





Let
$$y = 10^{x^{x}} + x^{x(10)} + x^{10x}$$

Put $u = 10^{x^{x}}, v = x^{x^{10}}$ and $\omega = x^{10^{x}}$
Then $y = u + v + \omega$
 $\therefore \frac{dy}{dx} = \frac{u}{dx} + \frac{dv}{dx} + \frac{d\omega}{dx}$...(1)
Take, $u = 10^{x^{x}}$
 $\therefore \frac{du}{dx} = \frac{d}{dx} (10^{x^{x}})$
 $z = 10^{x^{x}} \cdot \log 10. \frac{d}{dx} (x^{x})$
To find $\frac{d}{dx} (x^{x})$
Let $z = x^{\chi}$
 $\therefore \log z = \log x^{\chi} = \chi \log x$
Differentiating both sides w.r.t. x, we get
 $\frac{1}{z} \cdot \frac{dz}{dx} = \frac{d}{dx} (x \log x)$
 $= x \cdot \frac{d}{dx} (\log x) + (\log x) \cdot \frac{d}{dx} (x)$
 $= x \times \frac{1}{x} + (\log x)(1)$
 $\therefore \frac{dz}{dx} = z(1 + \log x)$
 $\therefore \frac{du}{dx} = 10^{x^{x}} \cdot \log 10. x^{x} (1 + \log x) ...(2)$
Take, $v = x^{x^{10}}$
 $\therefore \log v = \log x^{x^{10}} = x^{10} \cdot \log x$



$$\frac{1}{v} \cdot \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x} \left(x^{10} \log x \right)$$

$$= x^{10} \cdot \frac{\mathrm{d}}{\mathrm{d}x} (\log x) + (\log x) \cdot \frac{\mathrm{d}}{\mathrm{d}x} \left(x^{10} \right)$$

$$= x^{10} \times \frac{1}{x} + (\log x) \left(10x^9 \right)$$

$$\therefore \frac{\mathrm{d}v}{\mathrm{d}x} = v \left[x^9 + 10x^9 \log x \right]$$

$$\therefore \frac{\mathrm{d}v}{\mathrm{d}x} = x^{x^{10}} \cdot x^9 (1 + 10 \log x) \qquad \dots (3)$$
Also, $\omega = x^{10x}$

$$\therefore \log \omega = \log x^{10x} = 10^x \cdot \log x$$

Differentiating both sides w.r.t. x, we get

$$\begin{aligned} \frac{1}{\omega} \cdot \frac{d\omega}{dx} &= \frac{d}{dx} (10^x \cdot \log x) \\ &= 10^x \cdot \frac{d}{dx} (\log x) + (\log x) \cdot \frac{d}{dx} (10^x) \\ &= 10^x \times \frac{1}{x} + (\log x)(10^x \cdot \log 10) \\ &\therefore \frac{d\omega}{dx} &= \omega \left[\frac{10^x}{x} + 10^x \cdot (\log x)(\log 10) \right] \\ &\therefore \frac{d\omega}{dx} &= x^{10x} \cdot 10^x \left[\frac{1}{x} + (\log x)(\log 10) \right] \dots (4) \end{aligned}$$

From (1),(2),(3) and (4), we get
$$\begin{aligned} \frac{dy}{dx} &= 10^{x \cdot x} \cdot \log 10 \cdot x^x (1 + \log x) + x^{x^{10}} \cdot x^9 (1 + 10 \log x) + x^{10x} \cdot 10^x \left[\frac{1}{x} + (\log x)(\log 10) \right]. \end{aligned}$$

Exercise 1.3 | Q 2.8 | Page 40

Differentiate the following w.r.t. x : $\left[(\tan x)^{\tan x} \right]^{\tan x}$ at $x = rac{\pi}{4}$

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Let $y = \left[(\tan x)^{\tan x} \right]^{\tan x}$ $\therefore \log y = \log[(\tan x)^{\tan x}] \tan x$ = tanx. log(tanx)^{tanx} = tanx. tanx log(tan x) = $(tanx)^2$. log(tan x) Differentiating both sides w.r.t. x, we get $\frac{1}{u} \cdot \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{d}}{\mathrm{dx}} [\tan x)^2 \cdot \log(\tan x)$ $= (\tan x)^2 \cdot \frac{\mathrm{d}}{\mathrm{d}x} (\log \tan x) + (\log \tan x) \cdot \frac{\mathrm{d}}{\mathrm{d}x} (\tan x)^2$ $= (\tan x)^2 \cdot \times \frac{1}{\tan x} \cdot \frac{\mathrm{d}}{\mathrm{d}x} (\tan x) + (\log \tan x) \times 2 \tan x \cdot \frac{\mathrm{d}}{\mathrm{d}x} (\tan x)$ $= (\tan x)^2 \times \frac{1}{\tan x} \cdot \sec^2 x + (\log \tan x) \times 2 \tan x \sec^2 x$ $\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = y \big[(\tan x) \big(\sec^2 x \big) + (\log \tan x) \big(2 \tan x \sec^2 x \big) \big]$ = [(tanx)^{tanx}]^{tanx}.(tanxsec²x)[1 + 2logtanx] If $x = \frac{\pi}{4}$, then $\frac{\mathrm{dy}}{\mathrm{dx}} = \left[\left(\frac{\tan \pi}{4} \right)^{\tan \frac{\pi}{4}} \right]^{\tan \frac{\pi}{4}} \left(\tan \frac{\pi}{4} \sec^2 \frac{\pi}{4} \right) \left[1 + 2\log \tan \frac{\pi}{4} \right]$ $= \left[(1)^{1} \right]^{1} \cdot \left[1 \left(\sqrt{2} \right)^{2} \right] [1 + 2 \log 1]$ = 1 x 2 x 1 ...[: log 1 = 0] = 2.

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Exercise 1.3 | Q 3.01 | Page 40

Find
$$\frac{\mathrm{d}y}{\mathrm{d}x}$$
 if $\sqrt{x} + \sqrt{y} = \sqrt{a}$

SOLUTION

$$\sqrt{x} + \sqrt{y} = \sqrt{a}$$

Differentiating both sides w.r.t. x, we get
$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$$
$$\therefore \frac{1}{2\sqrt{y}} \frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$
$$\therefore \frac{dy}{dx} = -\sqrt{\frac{y}{x}}.$$

Exercise 1.3 | Q 3.02 | Page 40

Find
$$rac{\mathrm{d} \mathrm{y}}{\mathrm{d} \mathrm{x}}$$
 if $x\sqrt{x}+y\sqrt{y}=a\sqrt{a}$

SOLUTION

$$x\sqrt{x} + y\sqrt{y} = a\sqrt{a}$$

$$\therefore x^{\frac{3}{2}} + y^{\frac{3}{2}} = a^{\frac{3}{2}}$$

Differentiating both sides w.r.t. x, we get

$$\frac{3}{2} \cdot x^{\frac{1}{2}} + \frac{3}{2} \cdot y^{\frac{1}{2}} \frac{dy}{dx} = 0$$

$$\therefore \frac{3}{2} \cdot y^{\frac{1}{2}} \frac{dy}{dx} = -\frac{3}{2}x^{\frac{1}{2}}$$

$$\therefore \frac{dy}{dx} = \frac{-x^{\frac{1}{2}}}{y^{\frac{1}{2}}}$$

$$= -\sqrt{\frac{x}{y}}.$$

Exercise 1.3 | Q 3.03 | Page 40
Find
$$\frac{\mathrm{d}y}{\mathrm{d}x}$$
 if $x + \sqrt{xy} + y = 1$

$$x + \sqrt{xy} + y = 1$$

Differentiating both sides w.r.t. x, we get
$$1 + \frac{1}{2\sqrt{xy}} \cdot \frac{d}{dx}(xy) + \frac{dy}{dx} = 0$$
$$\therefore 1 + \frac{1}{2\sqrt{x}} \cdot \left[x\frac{dy}{dx} + y \times 1\right] + \frac{dy}{dx} = 0$$
$$\therefore 1 + \frac{1}{2}\sqrt{\frac{x}{y}}\frac{dy}{dx} + \frac{1}{2}\sqrt{\frac{y}{x}} + \frac{dy}{dx} = 0$$
$$\therefore \left(\frac{1}{2}\sqrt{\frac{x}{y}} + 1\right)\frac{dy}{dx} = -\frac{1}{2}\sqrt{\frac{y}{x}} - 1$$
$$\therefore \left(\frac{\sqrt{x} + 2\sqrt{y}}{2\sqrt{y}}\right)\frac{dy}{dx} = \frac{-\sqrt{y} - 2\sqrt{y}}{2\sqrt{x}}$$
$$\therefore \frac{dy}{dx} = \frac{-\sqrt{y}(2\sqrt{x} + \sqrt{y})}{\sqrt{x}(\sqrt{x} + 2\sqrt{y})}.$$

Exercise 1.3 | Q 3.04 | Page 40 Find $\frac{dy}{dx}$ If $x^3 + x^2y + xy^2 + y^3 = 81$





$$x^{3} + x^{2}y + xy^{2} + y^{3} = 81$$

Differentiating both sides w.r.t. x, we get

$$3x^{2} + x^{2}\frac{dy}{dx} + y\frac{d}{dx}(x^{2}) + x\frac{d}{dx}(y^{2}) + y^{2}\frac{d}{dx}(x) + 3y^{2}\frac{dy}{dx} = 0$$

$$\therefore 3x^{2} + x^{2}\frac{dy}{dx} + y \times 2x + x \times 2y\frac{dy}{dx} + y^{2} \times 1 + 3y^{2}\frac{dy}{dx} = 0$$

$$\therefore 3x^{2} + x^{2}\frac{dy}{dx} + 2xy + 2xy\frac{dy}{dx} + y^{2} + 3y^{2}\frac{dy}{dx} = 0$$

$$\therefore (x^{2} + 2xy + 3y^{2})\frac{dy}{dx} = -3x^{2} 2xy - y^{2}$$

$$\therefore \frac{dy}{dx} = \frac{(-3x^{2} + 2xy + 3y^{2})}{x^{2} + 2xy + 3y^{2}}.$$

Exercise 1.3 | Q 3.05 | Page 40

Find
$$\frac{\mathrm{d}y}{\mathrm{d}x}$$
 if $x^2y^2 - \tan^{-1}\left(\sqrt{x^2 + y^2}\right) = \cot^{-1}\left(\sqrt{x^2 + y^2}\right)$

SOLUTION

$$x^{2}y^{2} - \tan^{-1}\left(\sqrt{x^{2} + y^{2}}\right) = \cot^{-1}\left(\sqrt{x^{2} + y^{2}}\right)$$
$$\therefore x^{2}y^{2} - \tan^{-1}\left(\sqrt{x^{2} + y^{2}}\right) + \cot^{-1}\left(\sqrt{x^{2} + y^{2}}\right)$$
$$\therefore x^{2}y^{2} = \frac{\pi}{2} \qquad \dots \left[\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$$
Differentiating both sides w.r.t. x, we get
$$x^{2} \cdot \frac{d}{dx}(y^{2}) + y^{2} \cdot \frac{d}{dx}(x^{2}) = 0$$
$$\therefore x^{2} \times 2y \frac{dy}{dx} + y^{2} \times 2x = 0$$



$$\therefore 2x^2 y \frac{\mathrm{d}y}{\mathrm{d}x} = -2xy^2$$
$$\therefore x \frac{\mathrm{d}y}{\mathrm{d}x} = -y$$
$$\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{y}{x}.$$

Exercise 1.3 | Q 3.06 | Page 40 $r:=d \frac{dy}{dx}$ if $xe^y + ve^x = 1$

Find
$$\frac{d}{dx}$$
 if $xe^y + ye^x$

SOLUTION

$$xe^y + ye^x = 1$$

Differentiating both sides w.r.t. x, we get

$$\frac{\mathrm{d}}{\mathrm{dx}}(xe^{y}) + \frac{\mathrm{d}}{\mathrm{dx}}(ye^{x}) = 0$$

$$\therefore x. \frac{\mathrm{d}}{\mathrm{dx}}(e^{y}) + e^{y}. \frac{\mathrm{d}}{\mathrm{dx}}(x) + y. \frac{\mathrm{d}}{\mathrm{dx}}(e^{x}) + e^{x}. \frac{\mathrm{dy}}{\mathrm{dx}} = 0$$

$$\therefore x. e^{y} \frac{\mathrm{dy}}{\mathrm{dx}} + e^{y} \times 1 + y \times e^{x} + e^{x} \frac{\mathrm{dy}}{\mathrm{dx}} = 0$$

$$\therefore (e^{x} + xe^{y}) \frac{\mathrm{dy}}{\mathrm{dx}} = -e^{y} - ye^{x}$$

$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = -\left(\frac{e^{y} + ye^{x}}{e^{x} + xe^{y}}\right).$$

Exercise 1.3 | Q 3.07 | Page 40

Find
$$\frac{dy}{dx}$$
 if $e^{x+y} = \cos(x-y)$

$$e^{x+y} = \cos(x-y)$$

Differentiating both sides w.r.t. x, we get
$$e^{x+y} \cdot \frac{d}{dx}(x+y) = -\sin(x-y) \cdot \frac{d}{dx}(x-y)$$
$$\therefore e^{x+y}\left(1 + \frac{dy}{dx}\right) = -\sin(x-y) \cdot \frac{dy}{dx}(x-y)$$
$$\therefore e^{x+y} + e^{x+y} \cdot \frac{dy}{dx} = -\sin(x-y)\left(1 - \frac{dy}{dx}\right)$$
$$\therefore \left[e^{x+y} - \sin(x-y)\right] \frac{dy}{dx} = -\sin(x-y) - e^{x+y}$$
$$\therefore \frac{dy}{dx} = -\left[\frac{\sin(x-y) + e^{x+y}}{e^{x+y} - \sin(x-y)}\right] = \frac{\sin(x-y) + e^{x+y}}{\sin(x-y) - e^{x+y}}.$$

Exercise 1.3 | Q 3.08 | Page 40

Find
$$\frac{dy}{dx}$$
 if $\cos(xy) = x + y$

SOLUTION

$$\cos (xy) = x + y$$

Differentiating both sides w.r.t. x, we get
$$-\sin(xy) \cdot \frac{d}{dx}(xy) = 1 + \frac{dy}{dx}$$
$$\therefore -\sin(xy) \left[x \frac{dy}{dx} + y \frac{d}{dx}(x) \right] = 1 + \frac{dy}{dx}$$
$$\therefore -\sin(xy) \left[x \frac{dy}{dx} + y \times 1 \right] = 1 + \frac{dy}{dx}$$
$$\therefore -x \sin(xy) \frac{dy}{dx} - y \sin(xy) = 1 + \frac{dy}{dx}$$



$$\therefore -\frac{\mathrm{dy}}{\mathrm{dx}} - \sin(xy)\frac{\mathrm{dy}}{\mathrm{dx}} = 1 + y\sin(xy)$$
$$\therefore -[1 + x\sin(xy)]\frac{\mathrm{dy}}{\mathrm{dx}} = 1 + y\sin(xy)$$
$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{-[1 + y\sin(xy)]}{1 + x\sin(xy)}.$$

Exercise 1.3 | Q 3.09 | Page 40

Find
$$\frac{\mathrm{d}y}{\mathrm{d}x}$$
 if $e^{e^{x-y}} = \frac{x}{y}$

SOLUTION

$$e^{e^{x-y}} = \frac{x}{y}$$

$$\therefore e^{x-y} = \log\left(\frac{x}{y}\right) \quad ...[\because e^{\chi} = y \Rightarrow x = \log y]$$

$$\therefore e^{\chi-y} = \log x - \log y$$

Differentiating both sides w.r.t. x, we get

$$e^{x-y} \cdot \frac{d}{dx}(x-y) = \frac{1}{x} - \frac{1}{y}\frac{dy}{dx}$$

$$\therefore e^{x-y}\left(1 - \frac{dy}{dx}\right) = \frac{1}{x} - \frac{1}{y}\frac{dy}{dx}$$

$$\therefore e^{x-y} - e^{x-y}\frac{dy}{dx} = \frac{1}{x} - \frac{1}{y}\frac{dy}{dx}$$

$$\therefore \left(\frac{1}{y} - e^{x-y}\right)\frac{dy}{dx} = \frac{1}{x} - e^{x-y}$$

$$\left(\frac{1-ye^{x-y}}{y}\right)\frac{dy}{dx} = \frac{1-xe^{x-y}}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y(1-xe^{x-y})}{(x(1-ye^{x-y}))}.$$

Exercise 1.3 | Q 4.1 | Page 40

Show that $\frac{dy}{dx} = \frac{y}{x}$ in the following, where a and p are constants : $x^7 \cdot y^5 = (x + y)^{12}$

SOLUTION

$$x^{7} \cdot y^{5} = (x + y)^{12}$$

$$\therefore (\log x^{7} \cdot y^{5}) = \log(x + y)^{12}$$

$$\therefore \log x^{7} + \log y^{5} = \log(x + y)^{12}$$

$$\therefore 7\log x + 5\log y = 12\log(x + y)$$

Differentiating both sides w.r.t. x, we get

$$7 \times \frac{1}{x} + 5 \times \frac{1}{y} \cdot \frac{dy}{dx} = 12 \times \frac{1}{x + y} \cdot \frac{d}{dx}(x + y)$$

$$\therefore \frac{7}{x} + \frac{5}{y} \cdot \frac{dy}{dx} = \frac{12}{x + y} \cdot \left(1 + \frac{dy}{dx}\right)$$

$$\therefore \frac{7}{x} + \frac{5}{y} \cdot \frac{y}{dx} = \frac{12}{x + y} + \frac{12}{x + y} \cdot \frac{dy}{dx}$$

$$\therefore \left(\frac{5}{y} - \frac{12}{x + y}\right) \frac{dy}{dx} = \frac{12}{x + y} - \frac{7}{x}$$

$$\therefore \left[\frac{5x + 5y - 12y}{y(x + y)}\right] \frac{dy}{dx} = \frac{12x - 7x - 7y}{x(x + y)}$$

$$\therefore \left[\frac{5x - 7y}{y(x + y)}\right] \frac{dy}{dx} = \frac{5x - 7y}{x(x - y)}$$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x}.$$

Exercise 1.3 | Q 4.2 | Page 40 Show that $\frac{dy}{dx} = \frac{y}{x}$ in the following, where a and p are constants : $x^py^4 = (x + y)^{p+4}$, $p \in N$



 $x^{p}v^{4} = (x + y)^{p+4}$ Taking log $\log(x^{p}y^{4}) = \log(x + y)^{p+4}$ $\log x^{p} + \log y^{4} = (p + 4) \log(x + y)$ $p \log x + 4 \log y = (p + 4) \log(x + y)$ Differentiating both sides w.r.t. x, we get $p. \frac{d}{dx}\log x + 4. \frac{d}{dx}\log y = (p+4)\frac{d}{dx}\log(x+y)$ $\frac{p}{x} + 4\frac{1}{y}\frac{\mathrm{dy}}{\mathrm{dx}} = (p+4)\frac{1}{x+y}\left(1+\frac{\mathrm{dy}}{\mathrm{dx}}\right)$ $\frac{p}{4} + \frac{4}{y}\frac{dy}{dx} = \frac{(p+4)}{(x+y)} + \frac{p+4}{(x+y)}\frac{dy}{dx}$ $\frac{\mathrm{dy}}{\mathrm{dx}}\left[\frac{4}{y}-\frac{(p+4)}{(x+y)}\right]=\frac{p+4}{x+y}-\frac{p}{x}$ $\frac{\mathrm{dy}}{\mathrm{dx}} \left[\frac{4(x+y) - y(p+4)}{y(x+y)} \right] = \frac{x(p+4) - p(x+y)}{x(x+y)}$ $\frac{\mathrm{dy}}{\mathrm{dx}} \left[\frac{4x + 4y - py - 4y}{u(x+y)} \right] = \frac{px + 4x - px - py}{x(x+y)}$ $\frac{\mathrm{dy}}{\mathrm{dx}}\left[\frac{4x-py}{y}\right] = \frac{4x-py}{x}$ $\frac{\mathrm{d}\mathbf{y}}{\mathrm{d}\mathbf{x}} = \frac{y}{x}.$

Exercise 1.3 | Q 4.3 | Page 40 Show that $\frac{dy}{dx} = \frac{y}{x}$ in the following, where a and p are constants : $\sec\left(\frac{x^5 + y^5}{x^5 - y^5}\right) = a^2$





$$\sec\left(\frac{x^5 + y^5}{x^5 - y^5}\right) = a^2$$

$$\therefore \frac{x^5 + y^5}{x^5 - y^5} = \sec^{-1}(a^2) = k$$

$$\therefore x^5 + y^5 = kx^5 - ky^5$$

$$\therefore (1 + k)y^5 = (k - 1)x^5$$

$$\therefore \frac{y^5}{x^5} = \frac{k - 1}{k + 1}$$

$$\therefore \frac{y}{x} = \left(\frac{k - 1}{k + 1}\right)^{\frac{1}{5}}, \text{ a constant}$$

Differentiating both sides w.r.t. x, we get

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{y}{x}\right) = 0$$

$$\therefore \frac{x \cdot \frac{\mathrm{dy}}{\mathrm{dx}} - y \cdot \frac{\mathrm{d}}{\mathrm{dx}}(x)}{x^2} = 0$$

$$\therefore x \frac{\mathrm{dy}}{\mathrm{dx}} - y \times 1 = 0$$

$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{y}{x}.$$

Alternative Method :

$$\sec\left(\frac{x^{5} + y^{5}}{x^{5} - y^{5}}\right) = a^{2}$$

∴ $\frac{x^{5} + y^{5}}{x^{5} - y^{5}} = \sec^{-1}a^{2} = k$...(Say)
∴ $x^{5} + y^{5} = kx^{5} - ky^{5}$
∴ $(1 + k)y^{5} = (k - 1)x^{5}$

$$\therefore \frac{y^5}{x^5} = \frac{k-1}{k+1} \qquad \dots (1)$$
$$\therefore y^5 = k'x^5, \text{ where } k' = \frac{k-1}{k+1}$$

Differentiating both sides w.r.t. x, we get

$$5y^{4} \frac{dy}{dx} = k' \times 5x4$$

$$\therefore \frac{dy}{dx} = k' \cdot \frac{x^{4}}{y^{4}}$$

$$\therefore \frac{dy}{dx} = \left(\frac{k-1}{k+1}\right) \cdot \frac{x^{4}}{y^{4}}$$

$$= \frac{y^{5}}{x^{5}} \times \frac{x^{4}}{y^{4}} \qquad \dots [By (1)]$$

$$\therefore \frac{dy}{dx} = \frac{y}{x}.$$

Exercise 1.3 | Q 4.4 | Page 40 Show that $\frac{dy}{dx} = \frac{y}{x}$ in the following, where a and p are constants : $\tan^{-1}\left(\frac{3x^2 - 4y^2}{3x^2 + 4y^2}\right) = a^2$

SOLUTION

$$\tan^{-1}\left(\frac{3x^2 - 4y^2}{3x^2 + 4y^2}\right) = a^2$$

$$\therefore \frac{3x^2 - 4y^2}{3x^2 + 4y^2} = \tan^2 = k \quad \dots \text{(Say)}$$

$$\therefore 3x^2 - 4y^2 = 3kx^2 + 4ky^2$$

$$\therefore (4k + 4)y^2 = (3 - 3k)x^2$$

$$\therefore \frac{y^2}{x^2} = \frac{3 - 3k}{4k + 4}$$

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$$\therefore \frac{y}{x} = \sqrt{\frac{3-3k}{4k+4}}, \text{ a constant}$$

Differentiating both sides w.r.t. x, we get
$$\frac{d}{dx} \left(\frac{y}{x}\right) = 0$$

$$\therefore \frac{x \frac{dy}{dx} - y \cdot \frac{d}{dx}(x)}{x^2} = 0$$

$$\therefore x \frac{dy}{dx} - y \times 1 = 0$$

$$\therefore x \cdot \frac{dy}{dx} = y$$

$$\therefore \frac{dy}{dx} = \frac{y}{x}.$$

Exercise 1.3 | Q 4.5 | Page 40

Show that $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{y}{x}$ in the following, where a and p are constants : $\cos^{-1}\left(\frac{7x^4 + 5y^4}{7x^4 - 5y^4}\right) = \tan^{-1}a$

SOLUTION

$$\cos^{-1}\left(\frac{7x^4 + 5y^4}{7x^4 - 5y^4}\right) = \tan^{-1}a$$

$$\frac{7x^4 + 5y^4}{7x^4 - 5y^4} = \cos(\tan^{-1}a) = b$$

$$\frac{7x^4 + 5y^4}{7x^4 - 5y^4} = b$$

$$7x^4 + 5y^4 = b(7x^4 - 5y^4)$$

$$7x^4 + 5y^4 = 7bx^4 - 5by^4$$

$$5y^4 + 5by^4 = 7bx^4 - 7x^4$$

$$5y^4(1 + b) = 7x^4(b - 1)$$
$$\frac{5y^4}{7x^4} = \frac{b-1}{1+b}$$
$$\frac{y^4}{x^4} = \frac{7(b-1)}{5(1+b)} = x$$
$$\frac{y^4}{x^4} = c....(1)$$
$$y^4 = cx^4$$

Differentiating both sides w.r.t. x, we get

$$4. y^{3} \frac{dy}{dx} = c.4x^{3}$$

$$\frac{dy}{dx} = \frac{c.4x^{3}}{4y^{3}}$$

$$\frac{dy}{dx} = \frac{c.x^{3}}{y^{3}}$$

$$\frac{dy}{dx} = \frac{y^{4}}{x^{4}} \cdot \frac{x^{3}}{y^{3}} \quad ... \text{from..(1)}$$

$$\frac{dy}{dx} = \frac{y}{x}.$$

Exercise 1.3 | Q 4.6 | Page 40 Show that $\frac{dy}{dx} = \frac{y}{x}$ in the following, where a and p are constants : $\log\left(\frac{x^{20} - y^{20}}{x^{20} + y^{20}}\right) = 20$





$$\log\left(\frac{x^{20} - y^{20}}{x^{20} + y^{20}}\right) = 20$$

$$\therefore \frac{x^{20} - y^{20}}{x^{20} + y^{20}} = e^{20} = k \quad ...(Say)$$

$$\therefore x^{20} - y^{20} = kx^{20} + ky^{20}$$

$$\therefore (1 + k)y^{20} = kx^{20} + ky^{20}$$

$$\therefore \frac{y^{20}}{x^{20}} = \frac{1 - k}{1 + k}$$

$$\therefore \frac{y}{x} = \left(\frac{1 - k}{1 + k}\right)^{\frac{1}{20}}, \text{ a constant}$$

Differentiating both sides w.r.t. x, we get

$$\frac{d}{dx}\left(\frac{y}{x}\right) = 0$$

$$\therefore \frac{x \frac{dy}{dx} - y \cdot \frac{d}{dx}(x)}{x^2} = 0$$

$$\therefore x \frac{dy}{dx} - y \times 1 = 0$$

$$\therefore x \frac{dy}{dx} = y$$

$$\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x}.$$

Exercise 1.3 | Q 4.7 | Page 40 Show that $\frac{dy}{dx} = \frac{y}{x}$ in the following, where a and p are constants : $e^{\frac{x^7 - y^7}{x^7 + y^7}} = a$





$$e^{\frac{x^7 - y^7}{x^7 + y^7}} = a$$

$$\therefore \frac{x^7 - y^7}{x^7 + y^7} = \log a = k \quad \dots (Say)$$

$$\therefore x^7 - y^7 = kx^7 + ky^7$$

$$\therefore (1 + k)y^7 = (1 - k)x^7$$

$$\therefore \frac{y^7}{x^7} = \frac{1 - k}{1 + k}$$

$$\therefore \frac{y}{x} = \left(\frac{1 - k}{1 + k}\right)^{\frac{1}{7}}, \text{ a constant}$$
Differentiating both sides w.r.t. x, we get
$$\frac{d}{dx} \left(\frac{y}{x}\right) = 0$$

$$\therefore \frac{x \frac{dy}{dx} - y \cdot \frac{d}{dx}(x)}{x^2} = 0$$

$$dy$$

$$\therefore x \frac{\mathrm{dy}}{\mathrm{dx}} - y \times 1 = 0$$
$$\therefore x \frac{\mathrm{dy}}{\mathrm{dx}} = y$$
$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{y}{x}.$$

Exercise 1.3 | Q 4.8 | Page 40

Show that
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x}$$
 in the following, where a and p are constants : $\sin\left(\frac{x^3 - y^3}{x^3 + y^3}\right) = a^3$

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$$\sin\left(\frac{x^3 - y^3}{x^3 + y^3}\right) = a^3$$
$$\frac{x^3 - y^3}{x^3 + y^3} = \sin a^3 = b$$
$$\frac{x^3 - y^3}{x^3 + y^3} = b$$
$$x^3 - y^3 = b(x^3 + y^3)$$
$$x^3 - y^3 = bx^3 + by^3$$
$$x^3 - bx^3 = by^3 + y^3$$
$$x^3(1 - b) = y^3(b + 1)$$
$$\frac{y^3}{x^3} = \frac{1 - b}{1 + b} = e$$
$$\frac{y^3}{x^3} = c \quad \dots (1)$$
$$y^3 = cx^3$$
Differentiating both sides w.r.t. x, we get
$$3y^2 \frac{dy}{dx} = c.3x^2$$
$$\frac{y^2 dy}{dx} = cx^2$$
$$\frac{dy}{dx} c \frac{x^2}{y^2}$$
$$\frac{dy}{dx} = \frac{y^3}{x^3} \cdot \frac{x^2}{y^2} \quad \dots \text{from}(1)$$
$$\frac{dy}{dx} = \frac{y}{x}.$$

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Exercise 1.3 | Q 5.01 | Page 40

If log (x + y) = log(xy) + p, where p is a constant, then prove that $\frac{dy}{dx} = \frac{-y^2}{x^2}$.

SOLUTION

 $\log(x + y) = \log(xy) + p$ $\therefore \log(x + y) = \log x + \log y + p$ Differentiating both sides w.r.t. x, we get $\frac{1}{x+y} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(x+y) = \frac{1}{x} + \frac{1}{y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} + 0$ $\therefore \frac{1}{x+y} \left(1 + \frac{\mathrm{dy}}{\mathrm{dx}} \right) = \frac{1}{x} + \frac{1}{y} \cdot \frac{\mathrm{dy}}{\mathrm{dx}}$ $\therefore \frac{1}{x+y} + \frac{1}{x+y} \cdot \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{x} + \frac{1}{y} \cdot \frac{\mathrm{dy}}{\mathrm{dx}}$ $\therefore \left(\frac{1}{x+y} - \frac{1}{y}\right) \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{x} - \frac{1}{x+y}$ $\therefore \left[\frac{y - x - y}{u(x + y)} \right] \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{x + y - x}{x(x + y)}$ $\therefore \left| \frac{-x}{y(x+y)} \right| \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x(x+y)}$ $\therefore \left(-\frac{x}{y}\right) \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{y}{x}$ $\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{y^2}{x^2}.$

Exercise 1.3 | Q 5.02 | Page 40

If
$$\log_{10}\left(\frac{x^3-y^3}{x^3+y^3}\right) = 2$$
, show that $\frac{\mathrm{dy}}{\mathrm{dx}} = -\frac{99x^2}{101y^2}$

$$\log_{10}\left(\frac{x^3 - y^3}{x^3 + y^3}\right) = 2$$

$$\therefore \frac{x^3 - y^3}{x^3 + y^3} = 102 = 100$$

$$\therefore x^3 - y^3 = 110x^3 + 100y^3$$

$$\therefore 101y^3 = -99x^3$$

$$\therefore y^3 = \frac{-99}{101}x^3$$

Differentiating both sides wrt

Differentiating both sides w.r.t. x, we get

$$3y^2 rac{\mathrm{dy}}{\mathrm{dx}} = rac{-99}{101} imes 3x^2$$

 $\therefore \mathrm{dy}?\mathrm{dx} = -rac{99x^2}{101y^2}.$

Exercise 1.3 | Q 5.03 | Page 40
If
$$\log_5\left(\frac{x^4 + y^4}{x^4 - y^4}\right) = 2$$
, show that $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{12x^3}{13y^3}$.

$$\log_{5}\left(\frac{x^{4} + y^{4}}{x^{4} - y^{4}}\right) = 2$$

$$\frac{x^{4} + y^{4}}{x^{4} - y^{4}} = 5^{2}$$

$$\frac{x^{4} + y^{4}}{x^{4} - y^{4}} = 25$$

$$x^{4} + y^{4} = 25x^{4} - 25y^{4}$$

$$26y^{4} = 24x^{4}$$

Differentiating both sides w.r.t. x, we get

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$$26\frac{d}{dx}y^4 = 24\frac{d}{dx}x^4$$
$$26.4y^3.\frac{dy}{dx} = 24.4.x^3$$
$$26y^3.\frac{dy}{dx} = 24.x^3$$
$$\frac{dy}{dx} = \frac{24x^3}{26y^3}$$
$$\frac{dx}{dx} = \frac{12x^3}{13y^3}$$

Exercise 1.3 | Q 5.04 | Page 40

If
$$e^{x} + e^{y} = e^{x+y}$$
, then show that $\frac{dy}{dx} = -e^{y-x}$.

SOLUTION

 $e^{x} + e^{y} = e^{x+y}$...(1)

Differentiating both sides w.r.t. x, we get

$$e^{x} + e^{y} \cdot \frac{\mathrm{dy}}{\mathrm{dx}} = e^{x+y} \cdot \frac{\mathrm{d}}{\mathrm{dx}}(x+y)$$
$$\therefore e^{x} + e^{y} \cdot \frac{\mathrm{dy}}{\mathrm{dx}} = e^{x+y} \cdot \left(1 + \frac{\mathrm{dy}}{\mathrm{dx}}\right)$$
$$\therefore e^{x} + e^{y} \frac{\mathrm{dy}}{\mathrm{dx}} = e^{x+y} + e^{x+y} \frac{\mathrm{dy}}{\mathrm{dx}}$$
$$\therefore \left(e^{y} - e^{x+y}\right) \frac{\mathrm{dy}}{\mathrm{dx}} = e^{x+y} - e^{x}$$
$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{e^{x+y} - e^{x}}{e^{y} - e^{x+y}}$$

$$= \frac{e^{x} + e^{y} - e^{x}}{e^{y} - e^{x} - e^{y}} \quad ...[By (1)]$$
$$= \frac{e^{y}}{-e^{x}}$$
$$= -e^{y-x}.$$

Exercise 1.3 | Q 5.05 | Page 40

If
$$\sin^{-1}\left(\frac{x^5-y^5}{x^5+y^5}\right)=\frac{\pi}{6}$$
, show that $\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{x^4}{3y^4}$

SOLUTION

$$\sin^{-1}\left(\frac{x^5 - y^5}{x^5 + y^5}\right) = \frac{\pi}{6}$$

$$\therefore \frac{x^5 - y^5}{x^5 + y^5} = \sin \frac{\pi}{6} = \frac{1}{2}$$

$$\therefore 2x^5 - 2y^5 = x^5 + y^5$$

$$\therefore 3y^5 = x^5$$

Differentiating both sides w.r.t. x, we get
$$3 \times 5y^4 \frac{dy}{dy} = 5y^4$$

$$3 \times 5y^4 \frac{\mathrm{d}y}{\mathrm{d}x} = 5x^4$$

 $\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^4}{3y^4}.$

Exercise 1.3 | Q 5.06 | Page 40
If
$$x^y = e^{x-y}$$
, then show that $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\log x}{(1 + \log x)^2}$.

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SOLUTION

$$x^{y} = e^{x-y}$$

$$\therefore \log xy = \log e^{x-y}$$

$$\therefore y \log x = (x - y)\log e$$

$$\therefore y \log x = x - y \qquad \dots [\because \log e = 1]$$

$$\therefore y + y\log x = x \qquad \therefore y(1 + \log x) = x$$

$$\therefore y = \frac{x}{1 + \log x}$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} \left(\frac{x}{1 + \log x}\right)$$

$$= \frac{(1 + \log x) \cdot \frac{d}{dx}(x) - x \frac{d}{dx}(1 + \log x)}{(1 + \log x)^{2}}$$

$$= \frac{(1 + \log x) \cdot 1 - x(0 + \frac{1}{x})}{(1 + \log x)^{2}}$$

$$= \frac{1 + \log x - 1}{(1 + \log x)^{2}}$$

$$= \frac{\log x}{(1 + \log x)^{2}}.$$

Exercise 1.3 | Q 5.07 | Page 40 If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots \infty}}}$, then show that $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\sin x}{1 - 2y}$.

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SOLUTION

 $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + ...\infty}}}$ $\therefore y^{2} = \cos x + \sqrt{\cos x + \sqrt{\cos x + ...\infty}}$ $\therefore y^{2} = \cos x + y$ Differentiating both sides w.r.t. x, we get

$$2y rac{\mathrm{dy}}{\mathrm{dx}} = -\sin x + rac{\mathrm{dy}}{\mathrm{dx}}$$

 $\therefore (1 - 2y) rac{\mathrm{dy}}{\mathrm{dx}} = \sin x$
 $\therefore rac{\mathrm{dy}}{\mathrm{dx}} = rac{\sin x}{1 - 2y}.$

Exercise 1.3 | Q 5.08 | Page 40
If
$$y = \sqrt{\log x + \log x + \sqrt{\log x + ...\infty}}$$
, show that $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{x(2y-1)}$.

$$y = \sqrt{\log x + \log x + \sqrt{\log x + ...\infty}}$$

$$\therefore y^{2} = \log x + \sqrt{\log x + \sqrt{\log x + ...\infty}}$$

$$\therefore y^{2} = \log x + y$$

Differentiating both sides w.r.t. x, we get

$$2y. \frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$$

$$\therefore (2y - 1)\frac{dy}{dx} = \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{1}{x(2y - 1)}.$$

Exercise 1.3 | Q 5.09 | Page 40

If y =
$$x^{x^{x^{\cdot}}}$$
, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2}{x(1-\log y)}.$





Exercise 1.3 | Q 5.1 | Page 40

If
$$e^y = y^x$$
, then show that $rac{\mathrm{d} y}{\mathrm{d} x} = rac{\left(\log y
ight)^2}{\log y - 1}.$





$$e^{y} = y^{x}$$

$$\therefore \log e^{y} = \log y^{x}$$

$$\therefore y \log e = x \log y$$

$$\therefore y = x \log y \qquad ...[\because \log e = 1] ...(1)$$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = x \frac{d}{dx}(\log y) + (\log y) \cdot \frac{d}{dx}(x)$$

$$\therefore \frac{dy}{dx} = x \times \frac{1}{y} \cdot \frac{dy}{dx} + (\log y) \times 1$$

$$\therefore \frac{dy}{dx} = \frac{x}{y} \frac{dy}{dx} + \log y$$

$$\therefore \left(1 - \frac{x}{y}\right) \frac{dy}{dx} = \log y$$

$$\therefore \left(\frac{y - x}{y}\right) \frac{dy}{dx} = \log y$$

$$\therefore \left(\frac{y - x}{y}\right) \frac{dy}{dx} = \log y$$

$$\therefore \frac{dy}{dx} = \frac{y \log y}{y - x}$$

$$= \frac{y \log y}{y - \left(\frac{y}{\log y}\right)} \qquad ...[By (1)]$$

$$\therefore \frac{dy}{dx} = \frac{(\log y)^{2}}{\log y - 1}.$$

Alternative Method :

$$ey = yx$$

$$\therefore \log ey = \log yx$$

$$\therefore y \log e = x \log y$$

$$\therefore y = x \log y \qquad ...[\because \log e = 1]$$

$$\therefore x = \frac{y}{\log y}$$

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Differentiating both sides w.r.t. x, we get

$$\frac{\mathrm{dx}}{\mathrm{dy}} = \frac{\mathrm{d}}{\mathrm{dy}} \left(\frac{y}{\log y} \right)$$

$$= \frac{(\log y) \cdot \frac{\mathrm{d}}{\mathrm{dy}}(y) - y \cdot \frac{\mathrm{d}}{\mathrm{dy}}(\log y)}{(\log y)^2}$$

$$= \frac{(\log y) \times 1 - y \times \frac{1}{y}}{(\log y)^2}$$

$$= \frac{\log y - 1}{(\log y)^2}$$

$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{\left(\frac{\mathrm{dx}}{\mathrm{dy}}\right)} = \frac{(\log y)^2}{\log y - 1}.$$

EXERCISE 1.4 [PAGES 48 - 49]

Exercise 1.4 | Q 1.1 | Page 48
Find
$$\frac{dy}{dx}$$
 if x = at², y = 2at

x = at², y = 2at
Differentiating x and y w.r.t. x, we get

$$\frac{dx}{dt} = \frac{d}{dt}(at^{2}) = a\frac{d}{dt}(t^{2})$$
= a x 2t = 2at
and

$$\frac{dy}{dt} = \frac{d}{dt}(2at) = 2a\frac{d}{dt}(t)$$
= 2a x 1 = 2a





$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\left(\frac{\mathrm{dy}}{\mathrm{dt}}\right)}{\left(\frac{\mathrm{dx}}{\mathrm{dt}}\right)}$$
$$= \frac{2a}{2at}$$
$$= \frac{1}{t}$$

Exercise 1.4 | Q 1.2 | Page 48 Find $\frac{dy}{dx}$ if x = a cot θ , y = b cosec θ

SOLUTION

a cot
$$\theta$$
, y = b cosec θ
Differentiating x and y w.r.t. x, we get
 $\frac{dx}{d\theta} = a \frac{d}{d\theta} (\cot \theta) = a(-\csc^2\theta)$
= $-\csc^2\theta$
and
 $\frac{dy}{d\theta} = b \frac{d}{d\theta} (\csc \theta) = b(-\csc \theta \cot \theta)$
= $-b \csc \theta \cot \theta$
 $\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-b\csc \theta}{-a\csc^2\theta}$
= $\frac{b}{a} \cdot \frac{\cot \theta}{\csc \theta}$
= $\frac{b}{a} \times \frac{\cos \theta}{\sin \theta} \times \sin \theta$
= $\left(\frac{b}{a}\right) \cos \theta$.

Exercise 1.4 | Q 1.3 | Page 48

Find
$$rac{\mathrm{d}y}{\mathrm{d}x}$$
, if : x = $\sqrt{a^2+m^2}, y = \logig(a^2+m^2ig)$

SOLUTION

$$x = \sqrt{a^2 + m^2}, y = \log(a^2 + m^2)$$

Differentiating x and y w.r.t. x, we get

$$\frac{dx}{dm} = \frac{d}{dm} \left(\sqrt{a^2 + m^2}\right)$$

$$= \frac{1}{2\sqrt{a^2 + m^2}} \cdot \frac{d}{dm} (a^2 + m^2)$$

$$= \frac{1}{2\sqrt{a^2 + m^2}} \times (0 + 2m) = \frac{m}{\sqrt{a^2 + m^2}}$$
and

$$\frac{dy}{dm} = \frac{d}{dm} \left[\log(a^2 + m^2)\right]$$

$$= \frac{1}{a^2 + m^2} \cdot \frac{d}{dm} (a^2 + m^2)$$

$$= \frac{1}{a^2 + m^2} \times (0 + 2m) = \frac{2m}{a^2 + m^2}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dm}\right)}{\left(\frac{dx}{dm}\right)}$$

$$= \frac{\left(\frac{2m}{a^2 + m^2}\right)}{\left(\frac{m}{\sqrt{a^2 + m^2}}\right)}$$

$$= \frac{2}{\sqrt{a^2 + m^2}}.$$

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Exercise 1.4 | Q 1.4 | Page 48
Find
$$\frac{dy}{dx}$$
, if : x = sin θ , y = tan θ

x = sin
$$\theta$$
, y = tan θ
Differentiating x and y w.r.t. x, we get
 $\frac{dx}{d\theta} = \frac{d}{d\theta}(\sin\theta) = \cos\theta$
and
 $\frac{dy}{d\theta} = \frac{d}{d\theta}(\tan\theta) = \sec^2\theta$
 $\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}$
 $= \frac{\sec^2\theta}{\cos\theta}$
= sec³ θ .

Exercise 1.4 | Q 1.5 | Page 48 Find $\frac{dy}{dx}$, if : x = a(1 - cos θ), y = b(θ - sin θ)





 $x = a(1 - \cos\theta), y = b(\theta - \sin\theta)$ Differentiating x and y w.r.t. x, we get $rac{\mathrm{dx}}{\mathrm{d}\theta} = a rac{\mathrm{d}}{\mathrm{d}\theta} (1 - \cos \theta)$ $= a[0 - (-\sin \theta)] = a \sin \theta$ and $rac{\mathrm{d} \mathrm{y}}{\mathrm{d} \mathrm{e}} = b rac{\mathrm{d}}{\mathrm{d} \mathrm{e}} (\mathrm{e} - \sin \mathrm{e})$ $= b(1 - \cos \theta)$ $\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)}{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)}$ $=\frac{b(1-\cos\theta)}{a\sin\theta}$ $=rac{b imes \sin^2\left(rac{ heta}{2}
ight)}{a imes 2\sin\left(rac{ heta}{2}
ight)\cos\left(rac{ heta}{2}
ight)}$ $=\left(\frac{b}{a}\right)\tan\left(\frac{\theta}{2}\right).$

Exercise 1.4 | Q 1.6 | Page 48

Find
$$\frac{\mathrm{dy}}{\mathrm{dx}}$$
, if : x = $\left(t + \frac{1}{t}\right)$, $y = a\left(t + \frac{1}{t}\right)$, where a > 0, a \neq 1, t \neq 0.





$$\begin{aligned} x &= \left(t + \frac{1}{t}\right), y = a\left(t + \frac{1}{t}\right) \qquad \dots(1) \\ \text{Differentiating x and y w.r.t. x, we get} \\ \frac{dx}{dt} &= \frac{d}{dt} \left(t + \frac{1}{t}\right)^{a} \\ &= a\left(t + \frac{1}{t}\right)^{a-1} \cdot \frac{d}{dt} \left(t + \frac{1}{t}\right) \\ &= a\left(t + \frac{1}{t}\right)^{a-1} \cdot \left(1 - \frac{1}{t^2}\right) \\ \text{and} \\ \frac{dy}{dt} &= \frac{d}{dt} \left[a^{(t+\frac{1}{t})}\right] \\ &= a^{(t+\frac{1}{t})} \cdot \log a \cdot \frac{d}{dt} \left(t + \frac{1}{t}\right) \\ &= a^{(t+\frac{1}{t})} \cdot \log a \cdot \left(1 - \frac{1}{t^2}\right) \\ &\therefore \frac{dy}{dx} &= \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} \\ &= \frac{a^{(t+\frac{1}{t})} \cdot \log a \cdot \left(1 - \frac{1}{t^2}\right)}{a(t+\frac{1}{t})^{a-1} \cdot \left(1 - \frac{1}{t^2}\right)} \\ &= \frac{a^{t+\frac{1}{t}} \cdot \log a \cdot \left(t + \frac{1}{t}\right)^{a}}{a \cdot \left(t + \frac{1}{t}\right)^{a}} \end{aligned}$$



$$= \frac{y \log a. \left(\frac{t^2+1}{t}\right)}{\operatorname{ax}} \qquad \dots [\operatorname{By}(1)]$$
$$= \frac{y(t^2+1) \log a}{\operatorname{axt}}.$$

Exercise 1.4 | Q 1.7 | Page 48

Find
$$\frac{\mathrm{d}y}{\mathrm{d}x}$$
, if : $x = \cos^{-1}\left(\frac{2t}{1+t^2}\right), y = \sec^{-1}\left(\sqrt{1+t^2}\right)$

SOLUTION

$$x = \cos^{-1}\left(\frac{2t}{1+t^2}\right), y = \sec^{-1}\left(\sqrt{1+t^2}\right)$$

Put t = tan θ .

Then
$$\theta = \tan^{-1}t$$

$$\therefore x = \cos^{-1}\left(\frac{2\tan\theta}{1+\tan^{2}\theta}\right), y = \sec^{-1}\left(\sqrt{1+\tan^{2}\theta}\right)$$
$$\therefore x = \cos^{-1}(\sin 2\theta), y = \sec^{-1}\left(\sqrt{\sec^{2}\theta}\right)$$
$$\therefore x = \cos^{-1}\left[\cos\left(\frac{\pi}{2} - 2\theta\right)\right], y = \sec^{-1}(\sec\theta)$$
$$\therefore x = \frac{\pi}{2} - 2\theta, y = \theta$$
$$\therefore x = \frac{\pi}{2} - 2\tan^{-1}t, y = \tan^{-1}t$$
Differentiating x and y w.r.t. x, we get
$$\frac{dx}{dt} = \frac{d}{dt}\left(\frac{\pi}{2}\right) - 2\frac{d}{dt}(\tan^{-1}t)$$
$$= 0 - 2 \times \frac{1}{1+t^{2}}$$
$$= \frac{-2}{1+t^{2}}$$
and



$$\begin{aligned} \frac{\mathrm{d}\mathbf{y}}{\mathrm{d}\mathbf{t}} &= \frac{\mathrm{d}}{\mathrm{d}\mathbf{t}} \left(\tan^{-1} t \right) \\ &= \frac{1}{1+t^2} \\ &\therefore \frac{\mathrm{d}\mathbf{y}}{\mathrm{d}\mathbf{x}} = \frac{\left(\frac{\mathrm{d}\mathbf{y}}{\mathrm{d}\mathbf{t}}\right)}{\left(\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\mathbf{t}}\right)} \\ &= \frac{\left(\frac{1}{1+t^2}\right)}{\left(\frac{-2}{1+t^2}\right)} \\ &= -\frac{1}{2}. \end{aligned}$$

Exercise 1.4 | Q 1.8 | Page 48

 $\therefore x = 3\theta$ and $y = \theta$

Find
$$\frac{\mathrm{d}y}{\mathrm{d}x}$$
, if : $x = \cos^{-1}(4t^3 - 3t)$, $y = \tan^{-1}\left(\frac{\sqrt{1-t^2}}{t}\right)$.

SOLUTION

$$x = \cos^{-1} (4t^3 - 3t), y = \tan^{-1} \left(\frac{\sqrt{1 - t^2}}{t} \right)$$

Put t = cos θ .
Then θ = cos⁻¹t.
 $\therefore x = \cos^{-1}(4\cos^3\theta - 3\cos\theta),$
 $y = \tan^{-1} \left(\frac{\sqrt{1 - \cos^2\theta}}{\cos\theta} \right)$
 $\therefore x = \cos^{-1}(\cos 3\theta), y = \tan^{-1} \left(\frac{\sin \theta}{\cos \theta} \right) = \tan^{-1}(\tan \theta)$



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$$\therefore x = 3\cos^{-1}t \text{ and } y = \cos^{-1}t$$

Differentiating x and y w.r.t. x, we get

$$\frac{dx}{dt} = 3\frac{d}{dt}(\cos^{-1})$$

$$= 3 \times \frac{-1}{\sqrt{1-t^2}}$$

$$= \frac{-3}{\sqrt{1-t^2}}$$
and

$$\frac{dy}{dt} = \frac{d}{dt}(\cos^{-1}t)$$

$$= \frac{-1}{\sqrt{1-t^2}}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

$$= \frac{\left(\frac{-1}{\sqrt{1-t^2}}\right)}{\left(\frac{-3}{\sqrt{1-t^2}}\right)}$$

$$= \frac{1}{3}.$$
Alternative Method :
 $x = \cos^{-1}(4t^3 - 2t) t = \tan^{-1}\left(\sqrt{1-t^2}\right)$

x = cos⁻¹ (4t³ – 3t), t = tan⁻¹
$$\left(\frac{\sqrt{1-t^2}}{t}\right)$$

Put t = cos θ .
Then x = cos⁻¹(4cos³ θ – 3cos θ),





$$y = \tan^{-1} \left(\frac{\sqrt{1 - \cos^2 \theta}}{\cos \theta} \right)$$

$$\therefore x = \cos^{-1} (\cos 3\theta), y = \tan^{-1} \left(\frac{\sin \theta}{\cos \theta} \right) = \tan^{-1} (\tan \theta)$$

$$\therefore x = 3\theta, y = \theta$$

$$\therefore x = 3y$$

$$\therefore y = \frac{1}{3}x$$

$$\therefore \frac{dy}{dx} = \frac{1}{3} \frac{d}{dx}(x)$$

$$= \frac{1}{3} \times 1$$

$$= \frac{1}{3}.$$

Exercise 1.4 | Q 2.1 | Page 48

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Find
$$\frac{dy}{dx}$$
 if : x = cosec² θ , y = cot³ θ at $\theta = \frac{\pi}{6}$

SOLUTION

x = cosec²
$$\theta$$
, y = cot³ θ
Differentiating x and y w.r.t. θ , we get
 $\frac{dx}{d\theta} = \frac{d}{d\theta} (cosec\theta)^2 = 2cosec\theta. \frac{d}{d\theta} (cosec\theta)$
= 2cosec θ (- cosec θ cot θ)
= - 2cosec² θ cot θ
and
 $\frac{dy}{d\theta} = \frac{d}{d\theta} (cot \theta)^3 = 3 \cot^2 \theta. \frac{d}{d\theta} (cot \theta)$
= 3cot² $\theta.(-cosec^2\theta)$
= -3cot² $\theta.cosec^2\theta$

$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\left(\frac{\mathrm{dy}}{\mathrm{d\theta}}\right)}{\left(\frac{\mathrm{dx}}{\mathrm{d\theta}}\right)} = \frac{-3\cot^2\theta.\cos^2\theta}{-2\csc^2\theta.\cot\theta}$$
$$= \frac{3}{2}\cot\theta$$
$$\therefore \left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)_{\mathrm{at}\,\theta = \frac{\pi}{6}}$$
$$= \frac{3}{2}\cot\frac{\pi}{6}$$
$$= \frac{3\sqrt{3}}{2}.$$

Exercise 1.4 | Q 2.2 | Page 48
Find
$$\frac{dy}{dx}$$
 if : x = a cos³ θ , y = a sin³ θ at $\theta = \frac{\pi}{3}$

a
$$\cos^{3}\theta$$
, y = a $\sin^{3}\theta$
Differentiating x and y w.r.t. θ , we get
 $\frac{dx}{d\theta} = a \frac{d}{d\theta} (\cos \theta)^{3}$
= $a \times 3 \cos^{2} \theta$. $\frac{d}{d\theta} (\cos \theta)$
= $3a \cos^{2}\theta (-\sin \theta)$
= $-3a \cos^{2}\theta \sin \theta$
and
 $\frac{dy}{d\theta} = a \frac{d}{d\theta} (\sin \theta)^{3}$

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$$= a \times 3 \sin^2 \theta. \frac{d}{d\theta} (\sin \theta)$$

= $3a \sin^2 \theta \cos \theta$
 $\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}$
 $= \frac{3a \sin^2 \cos \theta}{-3a \cos^2 \theta \sin \theta}$
 $= -\tan \theta$
 $\therefore \left(\frac{dx}{dy}\right)_{at \theta - \frac{\pi}{3}}$
 $= -\tan \frac{\pi}{3}$
 $= -\sqrt{3}.$

Exercise 1.4 | Q 2.3 | Page 48
Find
$$\frac{dy}{dx}$$
 if : x = t² + t + 1, y = sin $\left(\frac{\pi t}{2}\right) + cos\left(\frac{\pi t}{2}\right)$ at $t = 1$





x = t² + t + 1, y =
$$\sin\left(\frac{\pi t}{2}\right) + \cos\left(\frac{\pi t}{2}\right)$$

Differentiating x and y w.r.t. t, we get

$$\frac{\mathrm{dx}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}} \left(t^2 + t + 1 \right)$$
$$= 2t + 1 + 0 = 2t + 1$$

and

$$\begin{aligned} \frac{\mathrm{dy}}{\mathrm{dt}} &= \frac{\mathrm{d}}{\mathrm{dt}} \left[\sin\left(\pi \frac{t}{2}\right) \right] + \frac{\mathrm{d}}{\mathrm{dt}} \left[\cos\left(\frac{\pi}{t}\right) \right] \\ &= \cos\left(\frac{\pi t}{2}\right) \cdot \frac{\mathrm{d}}{\mathrm{dt}} \left(\frac{\pi t}{2}\right) + \left[-\sin\left(\frac{\pi t}{2}\right) \right] \cdot \frac{\mathrm{d}}{\mathrm{dt}} \left(\frac{\pi t}{2}\right) \\ &= \cos\left(\frac{\pi t}{2}\right) \times \frac{\pi}{2} \times 1 - \sin\left(\frac{\pi t}{2}\right) \times \frac{\pi}{2} \times 1 \\ &= \frac{\pi}{2} \left[\cos\left(\frac{\pi t}{2}\right) - \sin\left(\frac{\pi t}{2}\right) \right] \\ &\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\left(\frac{\mathrm{dy}}{\mathrm{dt}}\right)}{\left(\frac{\mathrm{dx}}{\mathrm{dt}}\right)} \\ &= \frac{\frac{\pi}{2} \left[\cos\left(\frac{\pi t}{2}\right) - \sin\left(\frac{\pi t}{2}\right) \right] \\ &= \frac{\pi}{2} \left[\cos\left(\frac{\pi t}{2}\right) - \sin\left(\frac{\pi t}{2}\right) \right] \end{aligned}$$

$$\therefore \left(\frac{dx}{dy}\right)_{\text{at }t=1}$$

$$= \frac{\frac{\pi}{2} \left[\cos \frac{\pi}{2} - \sin \frac{\pi}{2}\right]}{2(1)+1}$$

$$= \frac{\frac{\pi}{2}(0-1)}{3}$$

$$= -\frac{\pi}{6}.$$



Exercise 1.4 | Q 2.4 | Page 48
Find
$$\frac{dy}{dx}$$
 if : x = 2 cot t + cos 2t, y = 2 sin t - sin 2t at t = $\frac{\pi}{4}$

2 cot t + cos 2t, y = 2 sin t - sin 2t Differentiating x and y w.r.t. t, we get $\frac{dx}{dt} = \frac{d}{dt}(2\cos t + \cos 2t)$ $= 2\frac{d}{dt}(\cos t) + \frac{d}{dt}(\cos 2t)$ $= 2(-\sin t) + (-\sin 2t) \cdot \frac{d}{dt}(2t)$ $= 2\sin t - \sin 2t \times 2 \times 1$ $= -2\sin t - 2\sin 2t$ and $\frac{dy}{dx} = \frac{d}{dt}(2\sin t - \sin 2t)$ $= 2\frac{d}{dt}(\sin t) - \frac{d}{dt}(\sin 2t)$ $= 2\cos t - \cos 2t \cdot \frac{d}{dt}(2t)$ $= 2\cos t - \cos 2t \times 2 \times 1$ $= 2\cos t - \cos 2t \times 2 \times 1$





$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\left(\frac{\mathrm{dy}}{\mathrm{dt}}\right)}{\left(\frac{\mathrm{dx}}{\mathrm{dt}}\right)}$$

$$= \frac{2\cos t - 2\cos 2t}{-2\sin t - 2\sin 2t}$$

$$= \frac{\cos t - \cos 2t}{-\sin t - \sin 2t}$$

$$\therefore \left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)_{\mathrm{at}\ t = \frac{\pi}{4}}$$

$$= \frac{\cos\frac{\pi}{4} - \cos\frac{\pi}{2}}{-\sin\frac{\pi}{4} - \sin\frac{\pi}{2}}$$

$$= \frac{\frac{1}{\sqrt{2}} - 0}{-\frac{1}{\sqrt{2}} - 1}$$

$$= \frac{-1}{1 + \sqrt{2}}$$

$$= \frac{-1}{1 + \sqrt{2}} + \frac{1 - \sqrt{2}}{1 - \sqrt{2}}$$

$$= 1 - \sqrt{2}.$$

Exercise 1.4 | Q 2.5 | Page 48
Find
$$\frac{dy}{dx}$$
 if : x = t + 2sin (π t), y = 3t - cos (π t) at t = $\frac{1}{2}$





 $x = t + 2sin (\pi t), y = 3t - cos (\pi t)$ Differentiating x and y w.r.t. t, we get $\frac{\mathrm{dx}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}} [t + 2\sin(\pi t)]$ $= \frac{\mathrm{d}}{\mathrm{d}t}(t) + 3. \frac{\mathrm{d}}{\mathrm{d}t}[\sin(\pi t)]$ $= 1 + 2 \times \cos(\pi t) \cdot \frac{\mathrm{d}}{\mathrm{d}\mathbf{x}}(\pi t)$ $= 1 + 2\cos(\pi t) \times \pi \times 1$ $= 1 + 2\pi \cos{(\pi t)}$ and $\frac{\mathrm{dy}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}} [3t - \cos(\pi t)]$ = $3 \times 1 - [-\sin(\pi t)]$. $\frac{\mathrm{d}}{\mathrm{dt}}(\pi t)$ $= 3 + \sin(\pi t) \times \pi \times 1$ $= 3 + \pi \sin(\pi t)$ $\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)}{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)}$ $=\frac{3+\pi\sin(\pi t)}{1+2\pi\cos(\pi t)}$ $\left. \begin{array}{c} \left(\displaystyle rac{dy}{dx}
ight)_{ ext{at } t = rac{1}{2}} \end{array}
ight.$ $=\frac{3+\sin\left(\frac{\pi}{2}\right)}{1+2\pi\cos\left(\frac{\pi}{2}\right)}$ $=\frac{3+\pi\times 1}{1+2\pi(0)}$ $= 3 + \pi$.





Exercise 1.4 | Q 3.1 | Page 48

$$|\mathsf{f} \mathsf{x} = a\sqrt{\sec \theta - \tan \theta}, y = a\sqrt{\sec \theta + \tan \theta}, \text{show that} \frac{\mathrm{d} \mathsf{y}}{\mathrm{d} \mathsf{x}} = \frac{y}{x}.$$

SOLUTION

 $x = a\sqrt{\sec \theta - \tan \theta}, y = a\sqrt{\sec \theta + \tan \theta}$ $\therefore \frac{x}{a} = \sqrt{\sec \theta - \tan \theta}, y = a\sqrt{\sec \theta + \tan \theta}$ $\therefore \sec \theta - \tan \theta = \frac{x^2}{a^2} \qquad ...(1)$ $\sec \theta + \tan \theta = \frac{y^2}{a^2} \qquad ...(2)$ Adding (1) and (2), we get $2\sec \theta = \frac{x^2}{a^2} + \frac{y^2}{a^2}$ $= \frac{x^2 + y^2}{a^2}$ $\therefore \sec \theta = \frac{x^2 + y^2}{2a^2}$ Subtracting (1) from (2), we get $2\tan \theta = \frac{y^2}{a^2} - \frac{x^2}{a^2}$ $= \frac{y^2 - x^2}{a^2}$





$$\therefore \tan \theta = \frac{y^2 - x^2}{2a^2}$$

$$\therefore \sec^2 \theta - \tan^2 \theta = 1 \text{ gives,}$$

$$\left(\frac{x^2 + y^2}{a^2}\right)^2 - \left(\frac{y^2 - x^2}{2a^2}\right)^2 = 1$$

$$\therefore (x^2 + y^2)^2 - (y^2 - x^2)^2 = 4a^2$$

$$\therefore (4 + 2x^2y^2 + y^4) - (y^4 - 2x^2y^2 + x^4) = 4a^4$$

$$\therefore 4x^2y^2 = 4a^4$$

$$\therefore x^2y^2 = a^4$$

$$\Rightarrow x^2y^2 = a^4$$

Differentiating both sides w.r.t. x, we get

$$x^{2} \cdot \frac{d}{dx}(y^{2}) + y^{2}\frac{d}{dx}(x^{3}) = 0$$

$$\therefore x^{2} \times 2y\frac{dy}{dx} + y62 \times 2x = 0$$

$$\therefore 2x^{2}y\frac{dy}{dx} = -2xy^{2}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x}.$$

Exercise 1.4 | Q 3.2 | Page 48

If $x = e^{\sin 3t}$, $y = e^{\cos 3t}$, then show that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.

SOLUTION

 $x = e^{\sin 3t}, y = e^{\cos 3t}$ $\therefore \log x = \log e^{\sin 3t}, \log y = \log e^{\cos 3t}$

 $\therefore \log x = (\sin 3t)(\log e), \log y = (\cos 3t)(\log e)$ $\therefore \log x = \sin 3t, \log y = \cos 3t \dots (1) \dots [\because \log e = 1]$

Differentiating both sides w.r.t. t, we get

$$\frac{1}{x} \cdot \frac{dx}{dt} = \frac{d}{dt} (\sin 3t) = \cos 3t \cdot \frac{d}{dt} (3t)$$

$$= \cos 3t \times 3 = 3 \cos 3t$$
and
$$\frac{1}{y} \cdot \frac{dy}{dt} = \frac{d}{dt} (\cos 3t) = -\sin 3t \cdot \frac{d}{dx} (3t)$$

$$= -\sin 3t \times 3 = 3 \cos 3t$$

$$\therefore \frac{dx}{dt} = 3x \cos 3t \text{ and } \frac{dy}{dt} = -3y \sin 3t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dx}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

$$= \frac{-3y \sin 3t}{3x \cos 3t}$$

$$= \frac{-y \sin 3t}{x \cos 3t}$$

$$= \frac{-y \log x}{x \log y} \cdot \cdots \cdot [By (1)]$$

Exercise 1.4 | Q 3.3 | Page 48

 $\mathsf{lf}\,\mathsf{x}=\frac{t+1}{t-1}, y=\frac{1-t}{t+1}, \text{then show that}\ y^2-\frac{\mathrm{dy}}{\mathrm{dx}}=\mathsf{0}.$

$$x = \frac{t+1}{t-1}, y = \frac{1-t}{t+1}$$

$$\therefore y = \frac{1}{\left(\frac{t+1}{1-t}\right)} = \frac{-1}{\left(\frac{t+1}{t-1}\right)}$$

$$\therefore y = -\frac{1}{x}$$

$$\therefore xy = -1 \qquad \dots(1)$$

Differentiating both sides w.r.t. t, we get

$$x\frac{dy}{dx} + \frac{d}{dx}(x) = 0$$

$$\therefore x\frac{dy}{dx} + y \times 1 = 0$$

$$\therefore -\frac{1}{y}\frac{dy}{dx} + y = 0 \qquad \dots [By (1)]$$

$$\therefore -\frac{dy}{dx} + y^{2} = 0$$

$$\therefore y^{2} - \frac{dy}{dx} = 0.$$

Exercise 1.4 | Q 3.4 | Page 48

If x = a cos³t, y = a sin³t, show that $\frac{\mathrm{dy}}{\mathrm{dx}} = -\left(\frac{y}{x}\right)^3$.

SOLUTION

$$x = a \cos^{3}t, y = a \sin^{3}t$$

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt} = a \frac{d}{dt} (\cos t)^{3} = a.3(\cos t)^{2} \frac{d}{dt} (\cos t)$$

$$= 3a\cos^{2}t(-\sin t) = -3a \cos^{2}t \sin t$$
and

$$\frac{dy}{dt} = a \frac{d}{dt} (\sin t)^{3}$$

$$= a.3(\sin t)^{2} \frac{d}{dt} (\sin t)$$

$$= 3a \sin^{2}t. \cos t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

$$= \frac{3a \sin^2 t \cos t}{-3a \cos^2 t \sin t}$$

$$= -\frac{\sin t}{\cos t} \qquad \dots(1)$$
Now, x = a cos³t
 $\therefore \cos^3 t = \frac{x}{a}$
 $\therefore \cos^3 t = \left(\frac{x}{a}\right)^{\frac{1}{3}}$
y = asin³t
 $\therefore \sin^3 t = \frac{y}{a}$
 $\therefore \cos^3 t = \left(\frac{y}{a}\right)^{\frac{1}{3}}$
 $\therefore from (1), \frac{dy}{dx} = -\frac{\frac{y^{\frac{1}{3}}}{a^{\frac{1}{3}}}}{\frac{x^{\frac{1}{3}}}{a^{\frac{1}{3}}}}$
 $= -\left(\frac{y}{x}\right)^{\frac{1}{3}}$.
Alternative Method :

x = a cos³t, y = a sin³t
∴ cos³ t =
$$\frac{x}{a}$$
, sin³ t = $\frac{y}{a}$





$$\therefore \cos t = \left(\frac{x}{a}\right)^{\frac{1}{3}}, \sin t = \left(\frac{y}{a}\right)^{\frac{1}{3}}$$
$$\therefore \cos^{2}t + \sin^{2}t = 1 \text{ gives}$$
$$\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{a}\right)^{\frac{2}{3}} = 1$$
$$\therefore x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$$

Differentiating both sides w.r.t. t, we get $2 + \frac{1}{2} + \frac{1}{2$

$$\frac{2}{3}x^{\frac{-1}{3}} + \frac{2}{3}y^{\frac{-1}{3}}, \frac{dy}{dx} = 0$$

$$\therefore \frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = -\frac{2}{3}x^{\frac{-1}{3}}$$

$$\therefore \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{-\frac{1}{2}} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}.$$

Exercise 1.4 | Q 3.5 | Page 48

If x =
$$2\cos^4(t + 3)$$
, y = $3\sin^{4(t + 3)}$, show that $\frac{dy}{dx} = -\sqrt{\frac{3y}{2x}}$.

SOLUTION

$$x = 2\cos^{4}(t+3), y = 3\sin^{4}(t+3)$$

$$\therefore \cos^{4}(t+3) = \frac{x}{2}, \sin^{4}(t+3) = \frac{y}{3}$$

$$\therefore \cos^{2}(t+3) = \sqrt{\frac{x}{2}}, \sin^{2}(t+3) = \sqrt{\frac{y}{3}}$$

$$\because \cos^{2}(t+3) + \sin^{2}(t+3) = 1$$

$$\therefore \sqrt{\frac{x}{2}} + \sqrt{\frac{y}{3}} = 1$$

Differentiating x and y w.r.t. t, we get

$$\frac{1}{\sqrt{2}} \frac{\mathrm{d}}{\mathrm{dx}} \left(\sqrt{x} \right) + \frac{1}{\sqrt{3}} \frac{\mathrm{d}}{\mathrm{dx}} \left(\sqrt{y} \right) = 0$$
$$\therefore \frac{1}{\sqrt{2}} \times \frac{1}{2\sqrt{x}} + \frac{1}{\sqrt{3}} \times \frac{1}{2\sqrt{y}} \cdot \frac{\mathrm{dy}}{\mathrm{dx}} = 0$$
$$\therefore \frac{1}{2\sqrt{3}} \frac{\mathrm{dy}}{\sqrt{y}} \cdot \frac{\mathrm{dy}}{\mathrm{dx}} = -\frac{1}{2\sqrt{2}} \frac{1}{\sqrt{x}}$$
$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = -\frac{\sqrt{3}}{\sqrt{2}} \frac{\sqrt{y}}{\sqrt{2}}$$
$$= -\sqrt{\frac{3y}{2x}}.$$

If x = log(1 + t²), y = t - tan⁻¹t, show that
$$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\sqrt{e^x - 1}}{2}$$
.

$$x = \log(1 + t^{2}), y = t - \tan^{-1}t$$

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt} = \frac{d}{dt} \left[\log(1 + t^{2}) \right]$$

$$= \frac{1}{1 + t^{2}} \cdot \frac{d}{dt} (1 - t^{2})$$

$$= \frac{1}{1 + t^{2}} \times (0 + 2t)$$

$$= \frac{2t}{1 + t^{2}}$$
and

$$\frac{dy}{dt} = \frac{d}{dt} (t) - \frac{d}{dt} (\tan^{-1} t)$$



$$= 1 - \frac{1}{1+t^2}$$

$$= \frac{1+t^2-1}{1+t^2}$$

$$= \frac{t^2}{1+t^2}$$

$$= \frac{t^2}{1+t^2}$$

$$= \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

$$= \frac{\left(\frac{t^2}{1+t^2}\right)}{\left(\frac{2t}{1+t^2}\right)}$$

$$= \frac{t}{2}$$
Now, x = log (1 + t^2)
$$\therefore 1 + t^2 = e^x$$

$$\therefore t^2 = e^x - 1$$

$$\therefore t = \sqrt{e^x - 1}$$

$$\therefore \frac{dy}{dx} = \frac{\sqrt{e^x - 1}}{2}$$

Exercise 1.4 | Q 3.7 | Page 48
If x = sin⁻¹(e^t), y =
$$\sqrt{1 - e^{2t}}$$
, show that sin $x + \frac{dy}{dx} = 0$




$$x = \sin^{-1}(e^{t}), y = \sqrt{1 - e^{2t}}$$

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt} = \frac{d}{dt} [\sin^{-1}(e^{1})]$$

$$= \frac{1}{\sqrt{1 - (e^{t})^{2}}} \cdot \frac{d}{dt} (e^{t})$$

$$= \frac{1}{\sqrt{1 - e^{2t}}} \times e^{2} = \frac{e^{2}}{\sqrt{1 - e^{2t}}}$$
and

$$\frac{dy}{dt} = \frac{d}{dt} (\sqrt{1 - e^{2}})$$

$$= \frac{1}{2\sqrt{1 - e^{2t}}} \cdot \frac{d}{dt} (1 - e^{2t})$$

$$= \frac{1}{2\sqrt{1 - e^{2t}}} \cdot \times (0 - e^{2t} \times 2)$$

$$= \frac{-e^{2t}}{\sqrt{1 - e^{2t}}}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

$$= \frac{\left(\frac{-e^{2t}}{\sqrt{1 - e^{2t}}}\right)}{\left(\frac{e^{t}}{\sqrt{1 - e^{2t}}}\right)}$$

$$= -e^{t}$$

$$= -\sin x \qquad ...[\because x = \sin^{-1}(e^{t})]$$

$$\therefore \sin x + \frac{dy}{dx} = 0.$$



Exercise 1.4 | Q 3.8 | Page 48

$$|f x = \frac{2bt}{1+t^2}, y = a\left(\frac{1-t^2}{1+t^2}\right), \text{show that} \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{b^2y}{a^2x}.$$

SOLUTION

$$x = \frac{2bt}{1+t^2}, y = a\left(\frac{1-t^2}{1+t^2}\right)$$

Put t = tan θ .
Then x = $b\left(\frac{2\tan\theta}{1+\tan\theta}\right), y = a\left(\frac{1-t^2}{1+\tan^2\theta}\right)$
 \therefore x = b sin 2 θ , y = a cos 2 θ
 $\therefore \frac{x}{b} = \sin 2\theta, \frac{y}{a} = \cos 2\theta$
 $\therefore \left(\frac{x}{b}\right)^2 + \left(\frac{y}{a}\right)^2 = \sin^2 2\theta + \cos^2 2\theta$
 $\therefore \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
Differentiating x and y w.r.t. y, we get
 $\frac{1}{b^2} \times 2x \frac{dx}{dy} + \frac{1}{a^2} \times 2y = 0$
 $\therefore \frac{2xdx}{b^2dy} = \frac{-2y}{a^2}$
 $\therefore \frac{dx}{dy} = -\frac{b^2y}{a^2x}.$

Exercise 1.4 | Q 4.1 | Page 49 DIfferentiate x sin x w.r.t. tan x.





Let $u = x \sin x$ and $v = \tan x$. Then we want to find $\frac{du}{dv}$ Differentiating u and v w.r.t. x, we get $\frac{du}{dx} = \frac{d}{dx}(x \sin x)$ $= x \frac{d}{dx}(\sin x) + (\sin x) \cdot \frac{d}{dx}(x)$ $= x \cos x + (\sin x) \times 1$ $= x \cos x + \sin x$ and $\frac{dv}{dx} = \frac{d}{dx}(\tan x) = \sec^2 x$ $\therefore \frac{du}{dv} = \frac{(\frac{du}{dx})}{(\frac{dv}{dx})}$ $= \frac{x \cos x + \sin x}{\sec^2 x}$.

Exercise 1.4 | Q 4.2 | Page 49
Differentiate
$$\sin^{-1}\left(\frac{2x}{1+x^2}\right)w.r.t.\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

SOLUTION

Let
$$u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$
 and
 $v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
Then we want to find $\frac{du}{dv}$.



Put x = tan
$$\theta$$
.
Then θ = tan⁻¹x.
u = sin⁻¹ $\left(\frac{2 \tan \theta}{1 + \tan \theta}\right)$
= sin⁻¹(sin2 θ)
= 2 θ
= 2tan⁻¹x
 $\therefore \frac{du}{dx} = 2 \frac{d}{dx} (\tan^{-1} x)$
= $2 \times \frac{1}{1 + x^2}$
= $\frac{2}{1 + x^2}$
Also, v = cos⁻¹ $\left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}\right)$
= cos⁻¹(cos 2 θ)
= 2 θ
= 2 tan⁻¹x
 $\therefore \frac{dv}{dx} = 2 \frac{d}{dx} (\tan^{-1} x)$
= $2 \times \frac{1}{1 + x^2}$
= $\frac{2}{1 + x^2}$





$$\therefore \frac{\mathrm{du}}{\mathrm{dv}} = \frac{\left(\frac{\mathrm{du}}{\mathrm{dx}}\right)}{\left(\frac{\mathrm{dv}}{\mathrm{dx}}\right)}$$
$$= \frac{\left(\frac{2}{1+x^2}\right)}{\left(\frac{2}{1+x^2}\right)}$$
$$= 1.$$

Alternative Method :

Let
$$u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$
 and $v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
Then we want to find $\frac{du}{dv}$
Put $x = \tan\theta$.
Then $u = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan\theta}\right)$
 $= \sin^{-1}\left(\sin 2\theta\right)$
 $= 2\theta$
and
 $v = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$
 $= \cos^{-1}\left(\cos 2\theta\right)$
 $= 2\theta$
 $\therefore u = v$
Differentiating both sides w.r.t. v, we get
 $\frac{du}{dv} = 1$.
Exercise 1.4 | Q.4.3 | Page 49
Differentiate $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)w.r.t. \sec^{-1}\left(\frac{1}{2x^2-1}\right)$.

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Let
$$u = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$
 and
 $v = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$.
Then we want to find $\frac{du}{dv}$.
Put $x = \cos\theta$.
Then $\theta = \cos^{-1}x$.
 $\therefore u = \tan^{-1}\left(\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}\right)$
 $= \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$
 $= \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\theta\right)\right]$
 $= \frac{\pi}{2} - \theta$
 $= \frac{\pi}{2} - \cos^{-1}x$
 $\therefore \frac{du}{dx} = \frac{d}{dx}\left(\frac{\pi}{2}\right) - \frac{d}{dx}\left(\cos^{-1}x\right)$
 $= 0 - \frac{-1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{-x^2}}$
 $v = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$
 $= \cos^{-1}(2\cos^2\theta - 1)$
 $= \cos^{-1}(\cos^2\theta)$



$$= 2\theta$$

$$= 2 \cos^{-1}x$$

$$\therefore \frac{dv}{dx} = 2 \cdot \frac{d}{dx} (\cos^{-1}x)$$

$$= \frac{-2}{\sqrt{1-x^2}}$$

$$\therefore \frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)}$$

$$= \frac{1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{-2}$$

$$= -\frac{1}{2} \cdot \frac{1}{2}$$

Exercise 1.4 | Q 4.4 | Page 49
Differentiate
$$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)w. r. t. \tan^{-1} x.$$

Let
$$u = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$
 and $v = \tan^{-1} x$.
Then we want to find $\frac{du}{dv}$.
Put $x = \tan\theta$.
Then $= \tan^{-1} x$.
 $\therefore u = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$
 $= \cos^{-1}(\cos 2\theta)$
 $= 2\theta$

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$$\therefore u = 2\tan^{-1}x$$

$$\therefore \frac{du}{dx} = 2 \cdot \frac{d}{dx} (\tan^{-1} x)$$

$$= 2 \times \frac{1}{1 + x^{2}}$$

$$= \frac{2}{1 + x^{2}}$$
Also, $v = \tan^{-1}x$

$$\therefore \frac{dv}{dx} = \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1 + x^{2}}$$

$$\therefore \frac{du}{dv} = \frac{\left(\frac{dy}{dx}\right)}{\left(\frac{dv}{dx}\right)}$$

$$= \frac{\left(\frac{2}{1 + x^{2}}\right)}{\left(\frac{1}{1 + x^{2}}\right)}$$

$$= 2.$$

Exercise 1.4 | Q 4.5 | Page 49

Differentiate 3^x w.r.t. log_x3.





Let $u = 3^x$ and $v = \log_x 3$. Then we want to find $\frac{du}{dw}$. Differentiating u and v w.r.t. x, we get $\frac{\mathrm{du}}{\mathrm{dv}} = \frac{\mathrm{d}}{\mathrm{dv}} (3^x)$ $= 3^{x}.log3$ and $\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}\mathbf{x}} = \frac{\mathrm{d}}{\mathrm{d}\mathbf{x}} (\log_x 3)$ $= \frac{\mathrm{d}}{\mathrm{d}\mathbf{x}} \left(\frac{\log 3}{\log r} \right)$ $= \log 3. \frac{\mathrm{d}}{\mathrm{d}x} (\log x)^{-1}$ $= (\log 3)(-1)(\log x)^{-2} \cdot \frac{\mathrm{d}}{\mathrm{dx}}(\log x)$ $= \frac{-\log 3}{\left(\log x\right)^2} \times \frac{1}{x}$ $= \frac{-\log 3}{x(\log x)^2}$ $\therefore \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}\mathbf{v}} = \frac{\left(\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}\mathbf{x}}\right)}{\left(\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}\mathbf{x}}\right)}$ $= \frac{3^x \cdot \log 3}{\left[\frac{-\log 3}{x(\log x)^2}\right]}$ $= -x(\log x)^2.3^{x}.$

Exercise 1.4 | Q 4.6 | Page 49

Differentiate
$$\tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right) w. r. t. \sec^{-1} x.$$

SOLUTION

Let $u = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ and $v = \sec^{-1} x$. Then we want to find $\frac{du}{dv}$. Differentiate u and v w.r.t. x, we get $\frac{\mathrm{du}}{\mathrm{dx}} = \frac{\mathrm{d}}{\mathrm{dx}} \left[\tan^{-1} \left(\frac{\cos}{1 + \sin x} \right) \right]$ $\frac{\cos x}{1+\sin x} = \frac{\sin\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}$ $=\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right).\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}$ $= \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$ $\therefore \frac{\mathrm{du}}{\mathrm{dv}} = \frac{\mathrm{d}}{\mathrm{dv}} \left[\tan^{-1} \tan \left(\frac{\pi}{4} - \frac{\pi}{2} \right) \right]$ $=\frac{\mathrm{d}}{\mathrm{d}\mathbf{x}}\left(\frac{\pi}{4}-\frac{x}{2}\right)$ $=\frac{\mathrm{d}}{\mathrm{d}\mathbf{x}}\left(\frac{\pi}{4}\right)-\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}\mathbf{x}}(x)$ $= 0 - \frac{1}{2} \times 1$ $=-\frac{1}{2}$ and $\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}\mathbf{x}} = \frac{\mathrm{d}}{\mathrm{d}\mathbf{x}} \left(\sec^{-1} x \right)$



$$= \frac{1}{x\sqrt{x^2 - 1}}$$
$$\therefore \frac{\mathrm{du}}{\mathrm{dx}} = \frac{\left(\frac{\mathrm{du}}{\mathrm{dx}}\right)}{\left(\frac{\mathrm{dv}}{\mathrm{dx}}\right)}$$
$$= \frac{\left(-\frac{1}{2}\right)}{\left(\frac{1}{x\sqrt{x^2 - 1}}\right)}$$
$$= -\frac{x\sqrt{x^2 - 1}}{2}.$$

Exercise 1.4 | Q 4.7 | Page 49

Differentiate x^x w.r.t. x^{six}.

SOLUTION

Let
$$u = x^x$$
 and $v = x^{sinx}$
Then we want to find $\frac{du}{dx}$.
Take, $u = x^x$
 $\therefore \log u \log x = x \log x$
Differentiating both sides w.r.t. x, we get
 $\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx}(x \log x)$
 $= x \frac{d}{dx}(\log x) + (\log x) \cdot \frac{d}{dx}(x)$
 $= x \times \frac{1}{x} + (\log x) \times 1$
 $\therefore \frac{du}{dx} = u(1 + \log x)$
 $= x^x(1 + \log x)$
Also, $v = x^{sinx}$



$$\therefore \log v = \log x^{\sin x} = (\sin x)(\log x)$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{v} \cdot \frac{dv}{dx} = \frac{d}{dx} [(\sin x)(\log x)]$$

$$= (\sin x) \cdot \frac{d}{dx} (\log x) + (\log x) \cdot \frac{d}{dx} (\sin x)$$

$$= (\sin x) \times \frac{1}{x} + (\log x)(\cos x)$$

$$\therefore \frac{dv}{dx} = v \left[\frac{\sin x}{x} + (\log x)(\cos x) \right]$$

$$= x^{\sin x} \left[\frac{\sin x}{x} + (\log x)(\cos x) \right]$$

$$\therefore \frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)}$$

$$= \frac{x^{x}(1 + \log x)}{x^{\sin x} [\frac{\sin x}{x} + (\log x)(\cos x)]}$$

$$= \frac{x(1 + \log x) \times x}{x^{\sin x} [\sin x + x \cos x \cdot \log x]}$$

$$= \frac{(1 + \log x) \cdot x^{x+1-\sin x}}{\sin x + x \cos x \cdot \log x}.$$

Exercise 1.4 | Q 4.8 | Page 49
Differentiate
$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)w.r.t \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right).$$





Let
$$u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$

and
 $v = \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$.
Then we want to find $\frac{du}{dv}$
 $u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$
Put $x = \tan\theta$.
Then $\theta = \tan^{-1}x$
and
 $\frac{\sqrt{1+x^2}-1}{x} = \frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}$
 $= \frac{\sec\theta-1}{\tan\theta}$





$$= \frac{\frac{1}{\cos\theta} - 1}{\left(\frac{\sin\theta}{\cos\theta}\right)}$$

$$= \frac{1 - \cos\theta}{\sin\theta}$$

$$= \frac{2\sin^2\left(\frac{\theta}{2}\right)}{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}$$

$$= \tan\left(\frac{\theta}{2}\right)$$

$$\therefore u = \tan^{-1}\left[\tan\left(\frac{\theta}{2}\right)\right] = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$$

$$\therefore \frac{du}{dx} = \frac{1}{2}\frac{d}{dx}(\tan^{-1}x)$$

$$= \frac{1}{2} \times \frac{1}{1 + x^2}$$

$$= \frac{1}{2(1 + x^2)}$$

$$v = \tan^{-1}\left(\frac{2x\sqrt{1 - x^2}}{1 - 2x^2}\right)$$
Put x = sin θ .
Then θ = sin⁻¹x
and

$$\frac{2x\sqrt{1 - x^2}}{1 - 2x^2}$$

$$= \frac{2\sin\theta\sqrt{1-\sin^2\theta}}{1-2\sin^2\theta}$$

$$= \frac{2\sin\theta\cos\theta}{1-2\sin^2\theta}$$

$$= \frac{\sin 2\theta}{1-2\sin^2\theta}$$

$$= \tan 2\theta$$

$$\therefore v = \tan^{-1}(\tan 2\theta)$$

$$= 2\theta$$

$$= 2\sin^{-1}x$$

$$\therefore \frac{dv}{dx} = 2\frac{d}{dx}(\sin^{-1}x)$$

$$= 2 \times \frac{1}{\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$$

$$\therefore \frac{dv}{dx} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)}$$

$$= \frac{\left[\frac{1}{2(1+x^2)}\right]}{\left(\frac{2}{\sqrt{1-x^2}}\right)}$$

$$= \frac{1}{2(1+x^2)} \times \frac{\sqrt{1-x^2}}{2}$$

$$= \frac{\sqrt{1-x^2}}{4(1+x^2)}.$$
EXERCISE 1.5 [PAGE 60]

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Exercise 1.5 | Q 1.1 | Page 60

Find the second order derivatives of the following : $2x^5 - 4x^3 - rac{2}{x^2} - 9$

SOLUTION

Let
$$y = 2x^5 - 4x^3 - \frac{2}{x^2} - 9$$

Then $\frac{dy}{dx} = \frac{d}{dx} \left(2x^5 - 4x^3 - \frac{2}{x^2} - 9 \right)$
 $= 2\frac{d}{dx} (x^5) - 4\frac{d}{dx} (x^3) - 2\frac{d}{dx} (x^{-2}) - \frac{d}{dx} (9)$
 $= 2 \times 5x^4 - 4 \times 3x^2 - 2(-2)x^{-3} - 0$
 $= 10x^4 - 12x^2 + 4x^{-3}$
and
 $\frac{d^2y}{dx^2} = \frac{d}{dx} (10x^4 - 12x^2 + 4x^{-3})$
 $= 10\frac{d}{dx} (x^4) - 12\frac{d}{dx} (x^2) + 4\frac{d}{dx} (x^{-3})$
 $= 10 \times 4x^3 - 12 \times 2x + 4(-3)x^{-4}$
 $= 40x^3 - 24x - \frac{12}{x^4}$.

Exercise 1.5 | Q 1.2 | Page 60 Find the second order derivatives of the following : e^{2x} . tan x





Let
$$y = e^{2x}$$
. tan x
Then $\frac{dy}{dx} = \frac{d}{dx} (e^{2x} \cdot \tan x)$
 $= e^{2x} \cdot \frac{d}{dx} (\tan x) + \tan x \cdot \frac{d}{dx} (e^{2x})$
 $= e^{2x} \cdot \sec^2 x + \tan x \times e^{2x} \cdot \frac{d}{dx} (2x)$
 $= e^{2x} \cdot \sec^2 x + e^{2x} \cdot \tan x \cdot 2$
 $= e^{2x} (\sec^2 x + 2\tan x)$
and
 $\frac{d^2y}{dx^2} = \frac{d}{dx} [e^{2x} (\sec^2 x + 2\tan x)]$
 $= e^{2x} \cdot \frac{d}{dx} (\sec^2 x + 2\tan x) + (\sec^2 x + 2\tan x) \frac{d}{dx} (e^{2x})$
 $= e^{2x} \left[\frac{d}{dx} (\sec x)^2 + 2 \frac{d}{dx} (\tan x) \right] + (\sec^2 x + 2\tan x) \times e^{2x} \cdot \frac{d}{dx} (2x)$
 $= e^{2x} \left[2 \sec x \cdot \frac{d}{dx} (\sec x) + 2 \sec^2 x \right] + (\sec^2 x + 2\tan x) e^{2x} \times 2$
 $= e^{2x} (2 \sec x \cdot \sec x \tan x + 2 \sec^2 x) + 2 e^{2x} (\sec^2 x + 2\tan x) e^{2x} \times 2$
 $= e^{2x} (\sec^2 x \tan x + \sec^2 x + \sec^2 x + 2\tan x)$
 $= 2e^{2x} (\sec^2 x \tan x + \sec^2 x + \sec^2 x + 2\tan x)$
 $= 2e^{2x} (\sec^2 x \tan x + \sec^2 x + \sec^2 x + 2\tan x)$
 $= 2e^{2x} [\sec^2 x (\tan x + 1) + 1 + \tan^2 x + 2\tan x]$
 $= 2e^{2x} [\sec^2 x (1 + \tan x) + (1 + \tan x)^2]$
 $= 2e^{2x} [(1 + \tan x) (\sec^2 x + 1 + \tan x)]$
 $= 2e^{2x} [(1 + \tan x) (2 + \tan x + \tan^2 x).$



Exercise 1.5 | Q 1.3 | Page 60 Find the second order derivatives of the following : e^{4x} . cos 5x

SOLUTION

Let
$$y = e^{4x} \cdot \cos 5x$$

Then $\frac{dy}{dx} = \frac{d}{dx} (e^{4x} \cdot \cos 5x)$
 $= e^{4x} \cdot \frac{d}{dx} (\cos 5x) + \cos 5x \cdot \frac{d}{dx} (e^{4x})$
 $= e^{4x} \cdot (-\sin 5x) \cdot \frac{d}{dx} (5x) + \cos 5x \times e^{4x} \cdot \frac{d}{dx} (4x)$
 $= -e^{4x} \cdot \sin 5x \cdot 5 + e^{4x} \cos 5x \times 4$
 $= e^{4x} (4 \cos 5x - 5 \sin 5x)$
and
 $\frac{d^2y}{dx^2} = \frac{d}{dx} [e^{4x} (4 \cos 5x - 5 \sin 5x)]$
 $= e^{4x} \frac{d}{dx} (4 \cos 5x - 5 \sin 5x) + (4 \cos 5x - 5 \sin 5x) \cdot \frac{d}{dx} (4x)$
 $= e^{4x} [4 - \sin 5x) \cdot \frac{d}{dx} (5x) - 5 \cos 5x \cdot \frac{d}{dx} (5x)] + (4 \cos 5x - 5 \sin 5x) \times e^{4x} \cdot \frac{d}{dx} (4x)$
 $= e^{4x} [-4 \sin 5x \cdot 5 - 5 \cos 5x \cdot 5] + (4 \cos 5x - 5 \sin 5x)e^{4x} \cdot 4$
 $= e^{4x} (-20 \sin 5x - 25 \cos 5x + 16 \cos 5x - 5 \sin 5x)$
 $= e^{4x} (-9 \cos 5x - 40 \sin 5x)$.

Exercise 1.5 | Q 1.4 | Page 60

Find the second order derivatives of the following : x³.logx



Let
$$y = x^3 \log x$$

Then, $\frac{dy}{dx} = \frac{d}{dx} (x^3 \cdot \log x)$
 $= x^3 \frac{d}{dx} (\log x) + (\log x) \cdot \frac{d}{dx} (x^3)$
 $= x^3 \times \frac{1}{x} + (\log x) \times 3x^2$
 $= x^2 + 3x^2 \log x$
 $= x^2(1 + 3 \log x)$
and
 $\frac{d^2 y}{dx^2} = \frac{d}{dx} [x^2(1 + 3 \log x)]$
 $= x^2 \cdot \frac{d}{dx} (1 + 3 \log x) + (1 + 3 \log x) \times 2x$
 $= x^2 \left(0 + 3 \times \frac{1}{x}\right) + (1 + 3 \log x) \times 2x$
 $= 3x + 2x + 6x \log x$
 $= 5x + 6x \log x$
 $= x(5 + 6 \log x).$

Exercise 1.5 | Q 1.5 | Page 60

Find the second order derivatives of the following : log(logx)





Let
$$y = \log(\log x)$$

Then $\frac{dy}{dx} = \frac{d}{dx} [\log(\log x)]$
 $= \frac{1}{\log x} \cdot \frac{d}{dx} (\log x)$
 $= \frac{1}{\log x} \times \frac{1}{x} = \frac{1}{x \log x}$
and
 $\frac{d^2 y}{dx^2} = \frac{d}{dx} (x \log x)^{-1}$
 $= (-1)(x \log x)^{-2} \cdot \frac{d}{dx} (x \log x)$
 $= \frac{-1}{(x \log x)^2} \cdot \left[x \frac{d}{dx} (\log x) + (\log x) \cdot \frac{d}{dx} (x) \right]$
 $= \frac{-1}{(x \log x)^2} \cdot \left[x \times \frac{1}{x} + (\log x) \times 1 \right]$
 $= -\frac{1 + \log x}{(x \log x)^2}.$

Exercise 1.5 | Q 2.1 | Page 60
Find
$$\frac{d^2y}{dx^2}$$
 of the following : x = a(θ – sin θ), y = a(1 – cos θ)





$$\begin{aligned} x &= a(\theta - \sin \theta), y = a(1 - \cos \theta) \\ \text{Differentiating x and y w.r.t. } \theta_{\text{i}} \text{ we get} \\ \frac{dx}{d\theta} &= a \frac{d}{d\theta} (\theta - \sin \theta) \\ &= a(1 - \cos \theta) \qquad \dots(1) \\ \text{and} \\ \frac{dy}{d\theta} &= a \frac{d}{d\theta} (1 - \cos \theta) \\ &= a[0 - (-\sin \theta)] \\ &= a \sin \theta \\ \therefore \frac{dy}{dx} &= \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} \\ &= \frac{a \sin \theta}{a(1 - \cos \theta)} \\ &= \frac{2 \sin\left(\frac{\theta}{2}\right) \cdot \cos\left(\frac{\theta}{2}\right)}{2 \sin^2\left(\frac{\theta}{2}\right)} = \cot\left(\frac{\theta}{2}\right) \\ \text{and} \\ \frac{d^2y}{dx^2} &= \frac{d}{dx} \left[\cot\left(\frac{\theta}{2}\right)\right] \\ &= \frac{d}{dx} \left[\cot\left(\frac{\theta}{2}\right)\right] \cdot \frac{d\theta}{dx} \right] \\ &= -\csc^2\left(\frac{\theta}{2}\right) \cdot \frac{d}{d\theta} \left(\frac{\theta}{2}\right) \times \frac{1}{\left(\frac{dx}{d\theta}\right)} \\ &= -\csc^2\left(\frac{\theta}{2}\right) \times \frac{1}{2} \times \frac{1}{a(1 - \cos \theta)} \quad \dots[by (1)] \end{aligned}$$





$$= -\frac{1}{2a} \operatorname{cosec}^{2} \left(\frac{\theta}{2} \right) \times \frac{1}{2 \sin^{2} \left(\frac{\theta}{2} \right)}$$
$$= -\frac{1}{4a} \cdot \operatorname{cosec}^{4} \left(\frac{\theta}{2} \right).$$

Exercise 1.5 | Q 2.2 | Page 60

Find
$$\frac{d^2y}{dx^2}$$
 of the following : x = 2at², y = 4at

SOLUTION

$$x = 2at^{2}, y = 4at$$

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt} = \frac{d}{dt} (2at^{2})$$

$$= 2a. \frac{d}{dt} (t^{2})$$

$$= 2a \times 2t = 4at$$
 ...(1)
and

$$\frac{dy}{dt} = \frac{d}{dt} (4at)$$

$$= 4a \frac{d}{dt} (t)$$

$$= 4a \times 1 = 4a$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

$$= \frac{4a}{4at} = \frac{1}{t}$$
and

$$\frac{d^{2}y}{dx^{2}} = \frac{d}{dt} \left(\frac{1}{t}\right)$$





$$= \frac{\mathrm{d}}{\mathrm{dt}} (t^{-1}) \times \frac{\mathrm{dt}}{\mathrm{dx}}$$

$$= 1(t)^{-2} \times \frac{1}{\left(\frac{\mathrm{dx}}{\mathrm{dt}}\right)}$$

$$= \frac{1}{t^2} \times \frac{1}{4at} \qquad \dots [\mathrm{By} (1)]$$

$$= -\frac{1}{4at^3}.$$

Exercise 1.5 | Q 2.3 | Page 60
Find
$$\frac{d^2y}{dx^2}$$
 of the following : x = sin θ , y = sin³ θ at $\theta = \frac{\pi}{2}$

$$x = \sin\theta, y = \sin^{3}\theta$$

Differentiating x and y w.r.t. θ , we get

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(\sin\theta) = \cos\theta \qquad ...(1)$$
and

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(\sin\theta)^{3}$$

$$= 3(\sin\theta)^{2} \cdot \frac{d}{d\theta}(\sin\theta)$$

$$= 3\sin^{2}\theta \cdot \cos\theta$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}$$

$$= \frac{3\sin^{2}\theta\cos\theta}{\cos\theta}$$

$$= 3\sin^{2}\theta$$

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and $rac{d^2 y}{d^2 - 2} = 3 rac{\mathrm{d}}{\mathrm{d} \mathrm{x}} (\sin \theta)^2$ $= 3 \frac{\mathrm{d}}{\mathrm{d}\theta} (\sin \theta)^2 \times \frac{\mathrm{d}\theta}{\mathrm{d}x}$ $= 3 \times 2 \sin \theta \frac{d}{d\theta} (\sin \theta) \times \frac{1}{\left(\frac{dx}{d\theta}\right)}$ $= 6 \sin \theta \cdot \cos \theta \times \frac{1}{\cos \theta}$...[By (1)] $= 6 \sin \theta$ $\therefore \left(rac{d^2y}{dx^2}
ight)_{\mathrm{at} heta=rac{\pi}{2}}$ $= 6 \sin \frac{\pi}{2}$ = 6 x 1 = 6. Alternative Method : $x = sin\theta$, $y = sin^3\theta$ $\therefore y = x^3$ $\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x} (x^3) = 3x^2$ $\therefore \frac{d^2 y}{dx^2} = 3 \frac{\mathrm{d}}{\mathrm{dx}} \left(x^2 \right)$ $= 3 \times 2 \times 2$ = 6xIf $\theta = \frac{\pi}{2}$, then $x = \sin \frac{\pi}{2} = 1$ $\therefore \left(\frac{d^2y}{dx^2}\right)_{\mathrm{at}\theta = \frac{\pi}{2}}$



$$= \left(\frac{d^2y}{dx^2}\right)_{\text{at}x = 1}$$
$$= 6(1)$$
$$= 6.$$

Exercise 1.5 | Q 2.4 | Page 60 Find $\frac{d^2y}{dx^2}$ of the following : x = a cos θ , y = b sin θ at $\theta = \frac{\pi}{4}$.

SOLUTION

$$x = a \cos \theta, y = b \sin \theta$$

Differentiating x and y w.r.t. θ , we get

$$\frac{dx}{d\theta} = \frac{d}{d\theta} (a \cos \theta)$$

$$= a \frac{d}{d\theta} (\cos \theta)$$

$$= a(-\sin \theta)$$

$$= -a \sin \theta \qquad ...(1)$$

and

$$\frac{dy}{d\theta} = \frac{d}{d\theta} (b \sin \theta)$$

$$= b \frac{d}{dx} (\sin \theta)$$

$$= b \cos \theta$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}$$

$$= \frac{b \cos \theta}{-a \sin \theta}$$





$$= \left(-\frac{b}{a}\right) \cot \theta$$

and
$$\frac{d^2 y}{dx^2} = \frac{d}{dx} \left[\left(-\frac{b}{a}\right) \cot \theta \right]$$

$$= -\frac{b}{a} \cdot \frac{d}{d\theta} (\cot \theta) \times \frac{d\theta}{dx}$$

$$= \left(-\frac{b}{a}\right) (-\csc^2 \theta) \times \frac{1}{\left(\frac{dx}{d\theta}\right)}$$

$$= \left(\frac{b}{a}\right) \csc^2 \theta \times \frac{1}{-a \sin \theta} \quad ..[By (1)]$$

$$= \left(-\frac{b}{a^2}\right) \csc^3 \theta$$

$$\therefore \left(\frac{d^2 y}{dx^2}\right)_{at\theta = \frac{\pi}{4}}$$

$$= \left(-\frac{b}{a^2}\right) \csc^3 \frac{\pi}{4}$$

$$= \frac{-b}{a^2} \times \left(\sqrt{2}\right)^3$$

$$= -\frac{2\sqrt{2b}}{a^2}.$$

Exercise 1.5 | Q 3.01 | Page 60
If x = at² and y = 2at, then show that
$$xy\frac{d^2y}{dx^2} + a = 0$$
.





 $x = at^2$ and y = 2at...(1) Differentiating x and y w.r.t. t, we get $\frac{\mathrm{dx}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}} \left(\mathrm{at}^2 \right)$ $=a\frac{\mathrm{d}}{\mathrm{dt}}(t^2)$ = a x 2t = 2at ...(2) and $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(2\mathrm{a}t)$ $=2a\frac{\mathrm{d}}{\mathrm{dt}}(t)$ = 2a x 1 = 2a $\therefore \frac{\mathrm{d} y}{\mathrm{d} x} = \frac{\left(\frac{\mathrm{d} y}{\mathrm{d} t}\right)}{\left(\frac{\mathrm{d} x}{\mathrm{d} t}\right)}$ $=\frac{2a}{2at}=\frac{1}{t}$ and $\frac{d^2y}{dr^2} = \frac{\mathrm{d}}{\mathrm{dx}} \left(\frac{1}{t}\right)$ $=\frac{\mathrm{d}}{\mathrm{d}t}(t^{-1}).\frac{\mathrm{d}t}{\mathrm{d}x}$ $= (-1)t^{-2} \cdot \frac{1}{\left(\frac{\mathrm{dx}}{\mathrm{dt}}\right)}$ $=\frac{-1}{t^2}\times\frac{1}{2at}$...[By (2)]

$$= \frac{1}{2at^{3}}.$$

$$\therefore \frac{d^{2}y}{dx^{2}} = -\frac{1}{(at^{2})(2at) \times a}$$

$$= -\frac{a}{xy} \qquad ...[By (1)]$$

$$\therefore xy \frac{d^{2}y}{dx^{2}} = -a$$

$$\therefore xy \frac{d^{2}y}{dx^{2}} + a = 0.$$

Exercise 1.5 | Q 3.02 | Page 60

If y =
$$e^{m \tan^{-1} x}$$
, show that $(1 + x^2) \frac{d^2 y}{dx^2} + (2x - m) \frac{dy}{dx} = 0$.

SOLUTION

$$y = e^{m \tan^{-1} x} \qquad \dots(1)$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} \left(e^{m \tan^{-1} x} \right)$$

$$= e^{m \tan^{-1} x} \cdot \frac{d}{dx} \left(m \tan^{-1} x \right)$$

$$= e^{m \tan^{-1} x} \times m \times \frac{1}{1 + x^{2}}$$

$$\therefore (1 + x^{2}) \frac{dy}{dx} = my \qquad \dots[By (1)]$$

Differentiaiting again w.r.t. x, we get

$$(1 + x^{2}) \cdot \frac{d}{dx} \left(\frac{dy}{dx} \right) + \frac{dy}{dx} \cdot \frac{d}{dx} \left(1 + x^{2} \right) = m \frac{dy}{dx}$$

$$\therefore (1 + x^{2}) \frac{d^{2}y}{dx^{2}} + \frac{dy}{dx} (0 + 2x) = m \frac{dy}{dx}$$

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$$\therefore (1+x^2)\frac{d^2y}{dx^2} + 2x. \frac{dy}{dx} = m\frac{dy}{dx}.$$
$$\therefore (1+x^2)\frac{d^2y}{dx^2} + (2x-m)\frac{dy}{dx} = 0.$$

Exercise 1.5 | Q 3.03 | Page 60

If x = cos t, y = e^{mt}, show that
$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{\mathrm{dy}}{\mathrm{dx}} - m^2y$$
 = 0.

SOLUTION

$$x = \cos t, y = e^{mt}$$

$$\therefore t = \cos^{-1}x \text{ and } y = e^{m\cos^{-1}x} \qquad \dots(1)$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} \left(e^{m\cos^{-1}x} \right)$$

$$= e^{m\cos^{-1}x} \cdot \frac{d}{dx} \left(m\cos^{-1}x \right)$$

$$= e^{m\cos^{-1}x} \times m \times \frac{-1}{\sqrt{1-x^2}}$$

$$\therefore \sqrt{1-x^2} \cdot \frac{dy}{dx} = -my \qquad \dots[By (1)]$$

$$\therefore \left(1-x^2\right) \left(\frac{dy}{dx}\right)^2 = m^2y^2$$

Differentiating again w.r.t. x, we get

$$\left(1-x^2\right) \cdot \frac{d}{dx} \left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2 \cdot \frac{d}{dx} \left(1-x^2\right) = m^2 \cdot \frac{d}{dx} \left(y^2\right)$$

$$\therefore (1-x^2) \cdot 2 \frac{\mathrm{dy}}{\mathrm{dx}} \cdot \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 (0-2x) = m^2 \times 2y \frac{\mathrm{dy}}{\mathrm{dx}}$$

Cancelling
$$2 \frac{\mathrm{d}y}{\mathrm{d}x}$$
 throughtout, we get



$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$$

$$\therefore (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0.$$

Exercise 1.5 | Q 3.04 | Page 60

If y = x + tan x, show that $\cos^2 x$. $\frac{d^2y}{dx^2} - 2y + 2x = 0$.

SOLUTION

$$y = x + \tan x$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(x + \tan x)$$

$$= 1 + \sec^{2}x$$

and

$$\frac{d^{2}y}{dx^{2}} = \frac{d}{dx}(1 + \sec x)^{2}$$

$$= \frac{d}{dx}(1) + \frac{d}{dx}(\sec x)^{2}$$

$$= 2\sec x \cdot \sec x \tan x$$

$$= 2 \sec^{2}x \tan x$$

$$\therefore \cos^{2}x \cdot \frac{d^{2}y}{dx^{2}} - 2y + 2x$$

$$= \cos^{2}x(2\sec^{2}x \tan x) - 2(x + \tan x) + 2x$$

$$= \cos^{2}x \times \frac{2}{\cos^{2}x} \times \tan x - 2x - 2\tan x + 2x$$

$$= 2 \tan x - 2 \tan x$$

$$\therefore \cos^{2}x \cdot \frac{d^{2}y}{dx^{2}} - 2y + 2x = 0.$$



Exercise 1.5 | Q 3.05 | Page 60

If $y = e^{ax}$.sin(bx), show that $y_2 - 2ay_1 + (a^2 + b^2)y = 0$.

SOLUTION

$$y = e^{ax} \sin(bx) \qquad \dots(1)$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} [e^{ax} \cdot \sin(bx)]$$

$$= e^{ax} \cdot \frac{d}{dx} [\sin(bx)] + \sin(bx) \cdot \frac{d}{dx} (e^{ax})$$

$$= e^{ax} \cdot \cos(bx) \cdot \frac{d}{dx} (bx) + \sin(bx) \times e^{ax} \cdot \frac{d}{dx} (ax)$$

$$= e^{ax} \cdot \cos(bx) \cdot \frac{d}{dx} (bx) + \sin(bx) \times e^{ax} \cdot \frac{d}{dx} (ax)$$

$$= e^{ax} \cdot \cos(bx) \times b + e^{ax} \cdot \sin(bx) \times a$$

$$\therefore y_1 = e^{ax} [b \cos(bx) + a \sin(bx)] \dots (2)$$
Differentiating again w.r.t. x, we get
$$\frac{dy_1}{dx} = \frac{d}{r} dx [e^{ax} \{b \cos(bx) + a \sin(bx)\}]$$

$$= e^{ax} \cdot \frac{d}{dx} [b \cos(bx) + a \sin(bx)] + [b \cos(bx) + a \sin(bx)] \cdot \frac{d}{dx} (e^{ax})$$

$$= e^{ax} \cdot \frac{d}{dx} [b \cos(bx) + a \sin(bx)] + [b \cos(bx) + a \sin(bx)] \cdot \frac{d}{dx} (e^{ax})$$

$$= e^{ax} \cdot \left[b\{-\sin(bx)\}, \frac{dy}{dx} (bx) + a \cos(bx), \frac{dy}{dx} (bx)\right] + [b \cos(bx) + a \sin(bx)] \times e^{ax} \cdot \frac{d}{dx} (ax)$$

$$= e^{ax} [-b \sin(bx) \times b + a \cos(bx) \times b\} + [b \cos(bx) + a \sin(bx)e^{ax} \times a$$

$$= e^{ax} [-b^{2}\sin(bx) + a b \cos(bx) + a^{2}\sin(bx)]$$

$$\therefore y_2 = e^{ax} [-b^{2}\sin(bx) + 2ab \cos(bx) + a^{2}\sin(bx)] - 2ae^{ax} [b \cos(bx) + a \sin(bx)] + (a^{2} + b^{2})e^{ax} \sin(bx) \dots [By (1), (2) and (3)]$$

$$= e^{ax} (a^{2} + b^{2})y = 0.$$

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Exercise 1.5 | Q 3.06 | Page 60

If
$$\sec^{-1}\left(\frac{7x^3-5y^3}{7^3+5y^3}\right) = m$$
, show $\frac{d^2y}{dx^2} = 0$.

SOLUTION

$$\sec^{-1}\left(\frac{7x^3 - 5y^3}{7^3 + 5y^3}\right) = m$$

$$\therefore \frac{7x^3 - 5y^3}{7^3 + 5y^3} = \sec m = k \quad ...(Say)$$

$$\therefore 7x^3 - 5y^3 = 7kx^3 + 5ky^3$$

$$\therefore (5k + 5)y^3 = (7 - 7k)x^2$$

$$\therefore \frac{y^3}{x^3} = \frac{7 - 7k}{5k + 5}$$

$$\therefore \frac{y}{x} = \left(\frac{7 - 7k}{5k + 5}\right)^{\frac{1}{3}} = p, \text{ where p is a constant}$$

$$\therefore \frac{d}{dx}\left(\frac{y}{x}\right) = \frac{d}{dx}(p)$$

$$\therefore \frac{x\frac{dy}{dx} - y\frac{d}{dx}(x)}{x^2} = 0$$

$$\therefore x\frac{dy}{dx} - y \times 1 = 0$$

$$\therefore x\frac{dy}{dx} = y$$





$$\therefore \frac{dy}{dx} = \frac{y}{x} \qquad \dots(1)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{y}{x}\right)$$

$$= \frac{x\frac{dy}{dx} - y\frac{d}{dx}(x)}{x^2}$$

$$= \frac{x\left(\frac{y}{x}\right) - y \times 1}{x^2} \qquad \dots[By (1)]$$

$$= \frac{y - y}{x^2}$$

$$= \frac{0}{x^2}$$

$$= 0$$
Note: $\frac{dy}{dx} = \frac{y}{x} \cdot \text{where } \frac{y}{x} = p.$

$$\therefore \frac{dy}{dx} = p, \text{ where } p \text{ is a constant.}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}(p) = 0.$$

Exercise 1.5 | Q 3.07 | Page 60 If $2y = \sqrt{x+1} + \sqrt{x-1}$, show that $4(x^2 - 1)y_2 + 4xy_1 - y = 0$.

SOLUTION

$$2y = \sqrt{x+1} + \sqrt{x-1}$$
 ...(1)

Differentiating both sides w.r.t. x, x we get

$$\therefore 2\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{d}}{\mathrm{dx}}\left(\sqrt{x+1}\right) + \frac{\mathrm{d}}{\mathrm{dx}}\left(\sqrt{x+1}\right)$$
$$= \frac{1}{2\sqrt{x+1}}(1+0) + \frac{1}{2\sqrt{x-1}}(1-0)$$

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$$\therefore 2\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{2\sqrt{x+1}} + \frac{1}{2\sqrt{x+1}}$$
$$= \frac{\sqrt{x+1} + \sqrt{x+1}}{2\sqrt{x+1}}$$
$$= \frac{2y}{2\sqrt{x+1}} \qquad \dots [\mathrm{By} (1)]$$
$$\therefore 2\sqrt{x^2 - 1} \frac{\mathrm{dy}}{\mathrm{dx}} = y$$
$$\therefore 4(x^2 - 1) \cdot \left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^2 = y^2$$

DIfferentiating both sides w.r.t. x, we get

$$4(x^{2}-1)\frac{d}{dx}\left(\frac{dy}{dx}\right)^{2} + \left(\frac{dy}{dx}\right)^{2} \cdot \frac{d}{dx}\left[4(x^{2}-1)\right] = 2y\frac{dy}{dx}$$
$$\therefore 4(x^{2}-1)\cdot 2\frac{dy}{dx}\cdot \frac{d2y}{dx^{2}} + \left(\frac{dy}{dx}\right)^{2}\cdot 4(2x) = 2y\left(\frac{dy}{dx}\right)$$
Cancelling $2\frac{dy}{dx}$ on both sides, we get
$$4(x^{2}-1)\frac{d^{2}y}{dx^{2}} + 4x\frac{dy}{dx} = y$$
$$\therefore 4(x^{2}-1)\frac{d^{2}y}{dx^{2}} + 4x\frac{dy}{dx} - y = 0$$
$$\therefore 4(x^{2}-1)y_{2} + 4xy_{1} - y = 0.$$

If y = log
$$\left(x + \sqrt{x^2 + a^2}\right)^m$$
, show that $\left(x^2 + a^2\right)\frac{d^2y}{dx^2} + x\frac{\mathrm{d}}{\mathrm{dx}} = 0$.

SOLUTION



$$y = \log\left(x + \sqrt{x^2 + a^2}\right)^m$$

$$= m \log\left(x + \sqrt{x^2 + a^2}\right)$$

$$\therefore \frac{dy}{dx} = m \frac{d}{dx} \left[\log\left(x + \sqrt{x^2 + a^2}\right)\right]$$

$$= m \times \frac{1}{x + \sqrt{x^2 + a^2}} \cdot \frac{d}{dx} \left(x + \sqrt{x^2 + a^2}\right)$$

$$= \frac{m}{x + \sqrt{x^2 + a^2}} \times \left[1 + \frac{1}{2\sqrt{x^2 + a^2}} \cdot \frac{d}{dx} \left(x^2 + a^2\right)\right]$$

$$= \frac{m}{x + \sqrt{x^2 + a^2}} \times \left[1 + \frac{1}{2\sqrt{x^2 + a^2}} \cdot (2x + 0)\right]$$

$$= \frac{m}{x + \sqrt{x^2 + a^2}} \times \frac{\sqrt{x^2 + a^2} + x}{\sqrt{x^2 + a^2}}$$

$$\therefore \frac{dy}{dx} = \frac{m}{\sqrt{x^2 + a^2}}$$

$$\therefore \sqrt{x^2 + a^2} \frac{dy}{dx} = m$$

$$\therefore (x^2 + a^2) \left(\frac{dy}{dx}\right)^2 = m^2$$

Differentiating both sides w.r.t. x, we get

$$(x^{2} + a^{2}) \cdot \frac{\mathrm{d}}{\mathrm{dx}} \left(\frac{\mathrm{d}y}{\mathrm{dx}}\right)^{2} + \left(\frac{\mathrm{d}y}{\mathrm{dx}}\right)^{2} \cdot \frac{\mathrm{d}}{\mathrm{dx}} \left(x^{2} + a^{2}\right) = \frac{\mathrm{d}}{\mathrm{dx}} \left(m^{2}\right)$$
$$\therefore \left(x^{2} + a^{2}\right) \times 2\frac{\mathrm{d}y}{\mathrm{dx}} \cdot \frac{\mathrm{d}}{\mathrm{dx}} \left(\frac{\mathrm{d}y}{\mathrm{dx}}\right) + \left(\frac{\mathrm{d}y}{\mathrm{dx}}\right)^{2} \times (2x + 0) = 0$$
$$\therefore \left(x^{2} + a^{2}\right) \cdot 2\frac{\mathrm{d}y}{\mathrm{dx}} \frac{\mathrm{d}^{2}y}{\mathrm{dx}^{2}} + 2x \left(\frac{\mathrm{d}y}{\mathrm{dx}}\right)^{2} = 0$$

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Cancelling
$$2 \frac{\mathrm{dy}}{\mathrm{dx}}$$
 throughtout, we get $(x^2 + a^2) \frac{d^2 y}{dx^2} + x \frac{\mathrm{dy}}{\prime} dx = 0.$

Exercise 1.5 | Q 3.09 | Page 60

If y = sin (m cos⁻¹x), then show that
$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$$
.

SOLUTION

y = sin (m cos⁻¹x)
∴ sin⁻¹ y = m cos⁻¹ x
Differentiating both sides w.r.t. x, we get

$$\frac{1}{\sqrt{1-y^2}} \cdot \frac{dy}{dx} = m \times \frac{-1}{\sqrt{1-x^2}}$$
∴ $\sqrt{1-x^2} \cdot \frac{dy}{dx} = -2\sqrt{1-y^2}$
∴ $(1-x^2)\left(\frac{dy}{dx}\right)^2 = m^2(1-y^2)$
∴ $(1-x^2)\left(\frac{dy}{dx}\right)^2 = m^2 - m^2y^2$
Differentiating both sides w.r.t. x, we get
 $(1-x^2) \cdot \frac{d}{dx}\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2 \cdot \frac{d}{dx}(1-x^2) = 0 - m^2 \cdot \frac{d}{dx}(y^2)$
∴ $(1-x^2) \cdot 2\frac{dy}{dx} \cdot \frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = -2m^2y\frac{dy}{dx}$
Cancelling $2\frac{dy}{dx}$ throughtout, we get

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$$(1-x^2)rac{d^2y}{dx^2} - xrac{\mathrm{dy}}{\mathrm{dx}} = -\mathrm{m}^2\mathrm{y}$$

 $\therefore (1-x^2)rac{d^2y}{dx^2} - xrac{\mathrm{dy}}{\mathrm{dx}} + m^2y = 0.$

Exercise 1.5 | Q 3.1 | Page 60 If $y = \log (\log 2x)$, show that $xy_2 + y_1 (1 + xy_1) = 0$.

SOLUTION

$$y = \log (\log 2x)$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} [\log(\log 2x)]$$

$$= \frac{1}{\log 2x} \cdot \frac{d}{dx} (\log 2x)$$

$$= \frac{1}{\log 2x} \times \frac{1}{2x} \cdot \frac{d}{dx} (2x)$$

$$= \frac{1}{\log 2x} \times \frac{1}{2x} \times 2$$

$$\therefore \frac{dy}{dx} = \frac{1}{x \log 2x}$$

$$\therefore (\log 2x) \cdot \frac{dy}{dx} = \frac{1}{x} \qquad ...(1)$$

Differentiating both sides w.r.t. x, we get

$$(\log 2x) \cdot \frac{d}{dx} \left(\frac{dx}{dy}\right) + \frac{dy}{dx} \cdot \frac{d}{dx} (\log 2x) = \frac{d}{dx} \left(\frac{1}{x}\right)$$

$$\therefore (\log 2x). rac{d^2y}{dx^2} + rac{\mathrm{dy}}{\mathrm{dx}}. rac{1}{2x}. rac{\mathrm{d}}{\mathrm{dx}}(2x) = -rac{1}{x^2}$$

$$\therefore (\log 2x) \cdot \frac{d^2y}{dx^2} + \frac{\mathrm{dy}}{\mathrm{dx}} \cdot \frac{1}{2x} \times 2 = -\frac{1}{x^2}$$

$$\therefore (\log 2x) \cdot \frac{d^2y}{dx^2} + \frac{1}{x} \cdot \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{x} \cdot \frac{1}{x}$$

$$\therefore (\log 2x) \cdot \frac{d^2y}{dx^2} + \left[(\log 2x) \cdot \frac{\mathrm{dy}}{\mathrm{dx}} \right] \frac{\mathrm{dy}}{\mathrm{dx}} = -\frac{1}{x} \left[(\log 2x) \cdot \frac{\mathrm{dy}}{\mathrm{dx}} \right] \quad \dots [By (1)]$$

Dividing throughout by log 2x, we get

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -\frac{1}{x}\frac{dy}{dx}$$
$$\therefore x\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = -\frac{dy}{dx}$$
$$\therefore x\frac{d^2y}{dx^2} + \frac{dy}{dx} + x\left(\frac{dy}{dx}\right)^2 = 0$$
$$\therefore x\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(1 + x\frac{dy}{dx}\right) = 0$$

 $\therefore xy_2 + y_1 (1 + xy_1) = 0.$

Exercise 1.5 | Q 3.11 | Page 60

If
$$x^2 + 6xy + y^2 = 10$$
, show that $\frac{d^2y}{dx^2} = \frac{80}{(3x+y)^3}$.

SOLUTION

$$x^2 + 6xy + y^2 = 10$$
 ...(1)

Differentiating both sides w.r.t. x, we get

$$2x + 6\left[x\frac{\mathrm{dy}}{\mathrm{dx}} + y.\frac{\mathrm{d}}{\mathrm{dx}}(x)\right] + 2y\frac{\mathrm{dy}}{\mathrm{dx}} = 0$$
$$\therefore 2x + 6x\frac{\mathrm{dy}}{\mathrm{dx}} + 6y \times 1 + 2y\frac{\mathrm{dy}}{\mathrm{dx}} = 0$$

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$$\therefore (6x+2y)\frac{\mathrm{d}y}{\mathrm{d}x} = -2x - 6y$$
$$\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2(x+3y)}{2(3x+y)} = -\left(\frac{x+3y}{3x+y}\right) \qquad \dots (2)$$

$$\therefore \frac{d^2 y}{dx^2} = -\frac{d}{dx} \left(\frac{x+3y}{3x+y} \right)$$

$$= -\left[\frac{(3x+y) \cdot \frac{d}{dx} (x+3y) - (x+3y) \cdot \frac{d}{dx} (3x+y)}{(3x+y)^2} \right]$$

$$= -\left[\frac{(3x+y) \left(1+3\frac{dy}{dx}\right) - (x+3y) \left(3+\frac{dy}{dx}\right)}{(3x+y)^2} \right]$$

$$= \frac{1}{(3x+y)^2} \left[-(3x+y) \left\{ 1 - \frac{3(x+3y)}{3x+y} \right\} + (x+3y) \left(3 - \frac{x+3y}{3x+y} \right) \right] \quad \dots [By (2)]$$

$$= \frac{1}{(3x+y)^2} \left[-(3x+y) \left[-(3x+y) \left(\frac{3x+y-3x-9y}{3x+y} \right) + (x+3y) \left(\frac{9x+3y-x-3y}{3x+y} \right) \right] \right]$$

$$= \frac{1}{(3x+y)^2} \left[8y + \frac{(x+3y)(8x)}{3x+y} \right]$$

$$= \frac{1}{(3x+y)^2} \left[\frac{(8y(3x+y)+(x+3y)8x)}{3x+y} \right]$$

$$= \frac{24xy+8y^2+8x^2+24xy}{(3x+y)^2}$$

$$= \frac{8x^2+48xy+8y^2}{(3x+y)^3}$$

$$= \frac{8(10)}{(3x+y)^3} \qquad \dots [By (1)]$$

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$$\therefore rac{d^2y}{dx^2} = rac{80}{\left(3x+y
ight)^3}$$

Exercise 1.5 | Q 3.12 | Page 60

If x = a sin t - b cos t, y = a cos t + b sin t, show that $\frac{d^2y}{dx^2} = -\frac{x^3 + y^2}{y^3}$.

SOLUTION

 $x = a \sin t - b \cos t$, $y = a \cos t + b \sin t$ Differentiating x and y w.r.t. t, we get $\frac{\mathrm{dx}}{\mathrm{dt}} = a \frac{\mathrm{d}}{\mathrm{dx}} (\sin t) - b \frac{\mathrm{d}}{\mathrm{dt}} (\cos t)$ $= a \cos t - b(-\sin t)$ = a cos t + b sin t and $\frac{\mathrm{dy}}{\mathrm{dt}} = a \frac{\mathrm{d}}{\mathrm{dx}} (\cos t) - b \frac{\mathrm{d}}{\mathrm{dt}} (\sin t)$ = a(-sint) + b cost= -a sin t + b cos t $\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)}{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)}$ $= \frac{-a \sin t + b \cos t}{a \cos t + b \sin t}$ $= -\left(\frac{a\sin t - b\cos t}{a\cos t + b\sin t}\right)$ $\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = -\frac{x}{y}$...(1)

$$\begin{split} & \therefore \frac{d^2 y}{dx^2} = -\frac{\mathrm{d}}{\mathrm{dx}} \left(\frac{x}{y}\right) \\ &= -\left[\frac{y \frac{\mathrm{d}}{\mathrm{dx}}(x) - x \frac{\mathrm{dy}}{\mathrm{dx}}}{y^2}\right] \\ &= -\left[\frac{y \times 1 - x \left(-\frac{x}{y}\right)}{y^2}\right] \qquad \dots [\mathrm{By}\ (1)] \\ &= -\left[\frac{y^2 + x^2}{y^3}\right] \\ & \therefore \frac{d^2 y}{dx^2} = -\frac{x^2 + y^2}{y^2}. \end{split}$$

Exercise 1.5 | Q 4.01 | Page 60

Find the n^{th} derivative of the following : $(ax + b)^m$

SOLUTION

Let
$$y = (ax + b)^m$$

Then $\frac{dy}{dx} = \frac{d}{dx}(ax + b)^m$
 $= m(ax + b)^{m-1} \cdot \frac{d}{dx}(ax + b)$
 $= m(ax + b)^{m-1} \cdot (a \times 1 + 0)$
 $= am(ax + b)^{m-1}$
 $\frac{d^2y}{dx^2} = \frac{d}{dx} \left[am(ax + b)^{m-1} \right]$
 $= am \frac{d}{dx} (ax + b)^{m-1}$

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$$= am(m-1)(ax+b)^{m-2} \cdot \frac{d}{dx}(ax+b)$$

$$= am(m-1)(ax+b)^{m-2} \times (a \times 1 + 0)$$

$$= a^{2}m(m-1)(ax+b)^{m-2}$$

$$\frac{d^{2}y}{dx^{3}} = \frac{d}{dx} \left[a^{2}m(m-1) \frac{d}{dx}(ax+b)^{m-2} \right]$$

$$= a^{2}m(m-1) \frac{d}{dx}(ax+b)^{m-2}$$

$$= a^{m}(m-1)(m-2)(ax+b)^{m-3} \frac{d}{dx}(ax+b)$$

$$= a^{2}m(m-1)(m-2)(ax+b)^{m-3} \times (a \times 1 + 0)$$

$$= a^{3}m(m-1)(m-2)(ax+b)^{m-3}$$
In general, the nth order derivative is given by
$$\frac{d^{n}y}{dx^{n}} = a^{n}m(m-1)(m-2) \dots (m-n+1)(ax+b)^{m-n}$$
Case (i) : if m > 0, m > n, then

$$\frac{d^{n}y}{dx^{n}} = \frac{(a^{n}.m(m-1)(m-2)...(m-n+1)(m-n)...3.2.1)}{(m-n)(m-n-1)...3.2.1} \times (ax+b)^{m-n}$$
$$\therefore \frac{d^{2}y}{dx^{n}} = \frac{(a^{n}.m!(ax+b)^{m-n})}{(m-n)!}, \text{If } m > 0, m > n.$$

Case (ii) : if m > 0 and m < n, then its mth order derivative is a constant and every derivatives after mth order are zero.

$$\therefore \frac{d^n y}{dx^n} = 0, \text{ if } m > 0, m = n.$$

Case (iii) : If m > 0, m = n, then

$$\frac{d^n y}{dx^n} = a^n \cdot n(n-1)(n-2) \dots (n-n+1)(ax+b)^{n-n}$$

= $a^n \cdot n(n-1)(n-2) \dots 1 \cdot (ax+b)^0$
 $\therefore \frac{d^n y}{dx^n} = an \cdot n!, \text{ if } m > 0, m = n.$





Exercise 1.5 | Q 4.02 | Page 60

Find the nth derivative of the following : $\frac{1}{x}$

SOLUTION

Let
$$y = \frac{1}{x}$$

Then $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{x}\right)$
 $= \frac{1}{x^2}$
 $= \frac{(-1)^1 1!}{x^2}$
 $\frac{d^2 y}{dx^2} = \frac{d}{dx}\left(-\frac{1}{x^2}\right)$
 $= 1\frac{d}{dx}(x^{-2})$
 $= (-1)(-2)x^{-3}$
 $= \frac{(-1)^{2\cdot1}\cdot2}{x^3}$
 $= \frac{(-1)^2 2!}{x^3}$
 $\frac{d^3 y}{dx^3} = \frac{d}{dx}\left[\frac{(-1)^2\cdot 2!}{x^3}\right]$





$$= (-1)^{2} \cdot 2! \frac{d}{dx} (x^{-3})$$
$$= (-1)^{2} \cdot 2! \cdot (-3) x^{-4}$$
$$= \frac{(-1)^{3} \times 3 \cdot 2!}{x^{4}}$$
$$= \frac{(-1)^{3} \cdot 3!}{x^{4}}$$

In general, the nth order derivative is given by

$$\frac{d^n y}{dx^n} = \frac{\left(-1\right)^n . \ n!}{x^{n+1}}.$$

Exercise 1.5 | Q 4.03 | Page 60

Find the nth derivative of the following : e^{ax+b}

SOLUTION

Let
$$y = e^{ax+b}$$

Then $\frac{dy}{dx} = \frac{d}{dx} (e^{ax+b})$
 $= e^{ax+b} \cdot \frac{d}{dx} (ax + b)$
 $= e^{ax+b} \times (a \times 1 + 0)$
 $= ae^{ax+b}$
 $\frac{d^2y}{dx^3} = \frac{d}{dx} (ae^{ax+b})$
 $= a \cdot \frac{d}{dx} (ax + b)$



$$= ae^{ax+b} \times (a \times 1 + 0)$$

$$= a^{2} \cdot e^{ax+b}$$

$$\frac{d^{3}y}{dx^{3}} = \frac{d}{dx} [a^{2}e^{ax+b}]$$

$$= a^{2} \frac{d}{dx} (e^{ax+b})$$

$$= a^{2}e^{ax+b} \cdot \frac{d}{dx} (ax+b)$$

$$= a^{2}e^{ax+b} \times (a \times 1 + 0)$$

$$= a^{3} \cdot e^{ax+b}$$

In genaral, the nth order derivative is given by

$$\frac{d^{n}y}{dx^{n}} = a^{n} \cdot e^{ax+b}.$$

Exercise 1.5 | Q 4.04 | Page 60

Find the nth derivative of the following : a^{px+q}

SOLUTION

Let
$$y = a^{px+q}$$

Then $\frac{dy}{dx} = \frac{d}{dx}(a^{px+q})$
 $= a^{px+q}\log a. \frac{d}{dx}(px+q)$
 $\frac{d^2y}{dx^2} = \frac{d}{dx}[p\log a. a^{px+q}]$
 $= p\log a. \frac{d}{dx}(a^{px+q})$



$$= p \log a. a^{px+q}. \log a. \frac{d}{dx} (px+q)$$

$$= p \log a. a^{px+q}. \log a \times (p \times 1+0)$$

$$= p^{2}. (\log a)^{2}. a^{px+q}$$

$$\frac{d^{3}y}{dx^{3}} = \frac{d}{dx} \left[p^{2}. (\log a)^{2}. a^{px+q} \right]$$

$$= p^{2}. (\log a)^{2}. \frac{d}{dx} (a^{px+q})$$

$$= p^{2}. (\log a)^{2}. a^{px+q}. \log a. \frac{d}{dx} (px+q)$$

$$= p^{2}. (\log a)^{3}. a^{px+q} \times (p \times 1+0)$$

$$= p^{3}. (\log a)^{3}. a^{px+q}$$
In general, the nth order derivative is given by
$$\frac{d^{n}y}{dx^{n}} = p^{n}. (\log a)^{n}. a^{px+q}.$$

Exercise 1.5 | Q 4.05 | Page 60

Find the n^{th} derivative of the following : log (ax + b)

Let
$$y = \log (ax + b)$$

Then $\frac{dy}{dx} = \frac{d}{dx} [\log(ax + b)]$
 $= \frac{1}{ax + b} \cdot \frac{d}{dx} (ax + b)$
 $= \frac{1}{ax + b} \times (a \times 1 + 0)$
 $= \frac{a}{ax + b}$
 $\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{a}{ax + b}\right)$

CLICK HERE



$$= a \frac{d}{dx} (ax + b)^{-1}$$

= $a(-1)(ax + b)^{-2} \cdot \frac{d}{dx}(ax + b)$
= $\frac{(-1)a}{(ax + b)^2} \times (a \times 1 + 0)$
= $\frac{(-1)a}{(ax + b)^2}$
 $\frac{d^3y}{dx^3} = \frac{d}{dx} \left[\frac{(-1)^1 a^2}{(ax + b)^2} \right]$
= $(-1)^1 a^2 \cdot \frac{d}{dx} (ax + b)^{-2}$
= $(-1)^1 a^2 \cdot (-2)(ax + b)^{-3} \cdot \frac{d}{dx}(ax + b)$
= $\frac{(-1)^2 \cdot 1 \cdot 2 \cdot a^2}{(ax + b)^3} \times (a \times 1 + 0)$
= $\frac{(-1) \cdot 2 \cdot 2! a^3}{(ax + b)^3}$

In general, the nth order derivative is given by $\frac{d^n y}{dx^2} = \frac{(-1)^{n-1} \cdot (n-1)! a^n}{(ax+b)^n}$

Exercise 1.5 | Q 4.06 | Page 60

Find the nth derivative of the following : cos x





Let
$$y = \cos x$$

Then $\frac{dy}{dx} = \frac{d}{dx}(\cos x)$
 $= -\sin x$
 $= \cos\left(\frac{\pi}{2} + x\right)$
 $\frac{d^2y}{dx^2} = \frac{d}{dx}(-\sin x)$
 $= -\cos x$
 $= \cos(\pi + x)$
 $= \cos\left(\frac{2\pi}{2} + x\right)$
 $\frac{d^3y}{dx^3} = \frac{d}{dx}(-\cos x)$
 $= -\frac{d}{dx}(\cos x)$
 $= -(-\sin x)$
 $= \sin x$
 $= \cos\left(\frac{3\pi}{2} + x\right)$

In general, the nth order derivative is given by $rac{d^n y}{dx^n} = \cos \Bigl(rac{n\pi}{2} + x \Bigr).$

Exercise 1.5 | Q 4.07 | Page 60

Find the n^{th} derivative of the following : sin (ax + b)

SOLUTION





Let y = sin (ax + b)
Then
$$\frac{dy}{dx} = \frac{d}{dx} [sin(ax + b)]$$

= $cos(ax + b) \cdot \frac{d}{dx} (ax + b)$
= $cos(ax + b) \times (a \times 1 + 0)$
= $a sin \left[\frac{\pi}{2} + (ax + b)\right]$
 $\frac{d^2y}{dx^2} = \frac{d}{dx} [a cos(ax + b)]$
= $a \frac{d}{dx} [cos(ax + b)]$
= $a \left[-sin(ax + b)\right] \cdot \frac{d}{dx} (ax + b)$
= $a \left[-sin(ax + b)\right] \cdot (a \times 1 + 0)$
= $a^2 \cdot sin \left[\pi + (ax + b)\right]$
= $a^2 \cdot sin \left[\frac{2\pi}{2} + (ax + b)\right]$
 $\frac{d^3y}{dx^3} = \frac{d}{dx} \left[-a^2 sin(ax + b)\right]$
= $-a^2 \frac{d}{dx} [sin(ax + b)]$
= $-a^2 \cdot cos(ax + b) \cdot \frac{d}{dx} (ax + b)$
= $-a^2 \cdot cos(ax + b) \cdot \frac{d}{dx} (ax + b)$
= $-a^2 \cdot cos(ax + b) \times (a \times 1 + 0)$
= $a^3 \cdot sin \left[\frac{3\pi}{2} + (ax + b)\right]$





In general, the nth order derivative is given by $rac{d^n y}{dx^n} = a^n . \sin \Bigl[rac{n\pi}{2} + (ax+b) \Bigr].$

Exercise 1.5 | Q 4.08 | Page 60

Find the n^{th} derivative of the following : cos (3 – 2x)

SOLUTION

Let
$$y = \cos (3 - 2x)$$

Then $\frac{dy}{dx} = \frac{d}{dx} [\cos(3 - 2x)]$
 $= \cos(3 - 2x) \cdot \frac{d}{dx} (3 - 2x)$
 $= \cos (3 - 2x) \times (a \times 1 + 0)$
 $= a \cos \left[\frac{\pi}{2} + (3 - 2x)\right]$
 $\frac{d^2 y}{dx^2} = \frac{d}{dx} [a \cos(3 - 2x)]$
 $= a \frac{d}{dx} [\cos(3 - 2x)]$
 $= a [-\cos(3 - 2x)] \cdot \frac{d}{dx} (3 - 2x)$
 $= a [-\cos(3 - 2x)] \times (a \times 1 + 0)$
 $= a^2 \cdot \cos[\pi + (3 - 2x)]$
 $= a^2 \cdot \cos\left[\frac{2\pi}{2} + (3 - 2x)\right]$





$$\frac{d^3y}{dx^3} = \frac{d}{dx} \left[-a^2 \cos(ax+b) \right]$$
$$= -a^2 \frac{d}{dx} \left[\cos(3-2x) \right]$$
$$= -a^2 \cdot \cos(3-2x) \cdot \frac{d}{dx} (3-2x)$$
$$= -a^2 \cdot \cos(3-2x) \times (a \times 1 + 0)$$
$$= a^3 \cdot \cos\left[\frac{3\pi}{2} + (3-2x) \right]$$
In general, the nth order derivative is given by

$$\frac{d^n y}{dx^n} = (-2)^n \cos\left[\frac{n\pi}{2} + (3-2x)\right].$$

Exercise 1.5 | Q 4.09 | Page 60

Find the n^{th} derivative of the following : log (2x + 3)

SOLUTION

Let
$$y = \log (2x + 3)$$

Then $\frac{dy}{dx} = \frac{d}{dx} [\log(2x + 3)]$
 $= \frac{1}{2x + 3} \cdot \frac{d}{dx} (2x + 3)$
 $= \frac{1}{2x + 3} \times (a \times 1 + 0)$
 $= \frac{a}{2x + 3}$
 $\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{a}{2x + 3}\right)$
 $= a \frac{d}{dx} (2x + 3)^{-1}$



$$= a(-1)(2x+3)^{-2} \cdot \frac{d}{dx}(2x+3)$$

$$= \frac{(-1)a}{(2x+3)^2} \times (a \times 1+0)$$

$$= \frac{(-1)a}{(2x+3)^2}$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx} \left[\frac{(-1)^1 a^2}{(2x+3)^2} \right]$$

$$= (-1)^1 a^2 \cdot \frac{d}{dx} (2x+3)^{-2}$$

$$= (-1)^1 a^2 \cdot (-2)(2x+3)^{-3} \cdot \frac{d}{dx} (2x+3)$$

$$= \frac{(-1)^2 \cdot 1 \cdot 2 \cdot a^2}{(2x+3)^3} \times (a \times 1+0)$$

$$= \frac{(-1)^2 \cdot 2 \cdot 2! a^3}{(2x+3)^3}$$

In general, the nth order derivative is given by $rac{d^n y}{dx^2} = rac{(-1)^{n-1}.\ (n-1)!2^n}{(2x+3)^n}.$

Exercise 1.5 | Q 4.1 | Page 60 Find the nth derivative of the following : $rac{1}{3x-5}$





Let
$$y = \frac{1}{3x - 5}$$

Then $\frac{dy}{dx} = \frac{d}{dx}(3x - 5)$
 $= -1(3x - 5)^{-2} \cdot \frac{d}{dx}(3x - 5)$
 $= \frac{-1}{(3x - 5)^2} \times (3 \times 1 - 0)$
 $= \frac{(-1)^{1} \cdot 3}{(3x - 5)^2}$
 $\frac{d^2y}{dx^2} = \frac{d}{dx} \left[\frac{(-1)^{1} \cdot 3}{(3x - 5)^2} \right]$
 $= (-1)^{1} \cdot 3 \frac{d}{dx}(3x - 5)^{-2}$
 $= (-1)^{-1} \cdot 3 \cdot (-2)(3x - 5)^{-3} \cdot \frac{d}{dx}(3x - 5)$
 $= \frac{(-1)^2 \cdot 3 \cdot 2}{(3x - 5)^3} \times (3 \times 1 - 0)$
 $\frac{d^3y}{dx^3} = \frac{d}{dx} \left[\frac{(-1)^2 \cdot 2! \cdot 3^2}{(3x - 5)^3} \right]$
 $= (-1)^2 \cdot 2! \cdot 3^2 \cdot \frac{d}{dx}(3x - 5)^{-3}$
 $= (-1)^2 \cdot 2! \cdot 3^2 \cdot (-3)(3x - 5)^{-4} \cdot \frac{d}{dx}(3x - 5)$
 $= \frac{(-1)^3 \times 3 \cdot 2! \times 3^2}{(3x - 5)^4} \times (3 \times 1 - 0)$



$$=rac{{{\left({-1}
ight)}^3} imes {3!} imes {3^3}}}{{{\left({3x - 5}
ight)}^4}}$$

In general, the nth order derivative is goven by $d^n w = (-1)^n m ! 2^n$

$$rac{d^n y}{dx^n} = rac{(-1)^n \cdot n! \cdot 3^n}{(3x-5)^{n+1}}.$$

Exercise 1.5 | Q 4.11 | Page 60

Find the nth derivative of the following : $y = e^{ax} \cdot \cos(bx + c)$

SOLUTION

$$y = e^{ax} \cdot \cos(bx + c)$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} [e^{ax} \cdot \cos[bx + c)]$$

$$= e^{ax} \cdot \frac{d}{dx} [\cos(bx + c)] + \cos(bx + c) \cdot \frac{d}{dx} (e^{ax})$$

$$= e^{ax} \cdot [-\sin(bx + c)] \cdot \frac{d}{dx} (bx + c) + \cos(bx + c) \cdot e^{ax} \cdot \frac{d}{dx} (ax)$$

$$= -e^{ax} \sin(bx + c) \times (b \times 1 + 0) + e^{ax} \cos(bx + c) \times a \times 1$$

$$= e^{ax} [a \cos(bx + c) - b \sin(bx + c)]$$

$$= e^{ax} \cdot \sqrt{a^2 + b^2} \left[\frac{a}{\sqrt{a^2 + b^2}} \cos(bx + c) - \frac{b}{\sqrt{a^2 + b^2}} \sin(bx + c) \right]$$

Let $\frac{a}{\sqrt{a^2 + b^2}} = \cos x$ and $\frac{b}{\sqrt{a^2 + b^2}} = \sin x$
Then $\tan \infty = \frac{b}{a}$
 $\therefore \infty = \tan^{-1} \left(\frac{b}{a}\right)$

$$\begin{split} \therefore \frac{dy}{dx} &= e^{ax} \cdot \sqrt{a^2 + b^2} [\cos \infty \cdot \cos(bx + c) - \sin \infty \cdot \sin(bx + c)] \\ &= e^{ax} \cdot (a^2 + b^2)^{\frac{1}{2}} \cdot \cos(bx + c + x) \\ \frac{d^2y}{dx^2} &= \frac{d}{dx} \left[e^{ax} \cdot (a^2 + b^2)^{\frac{1}{2}} \cdot \cos(bx + c + \infty) \right] \\ &= (a^2 + b^2)^{\frac{1}{2}} \left[e^{ax} \cdot (a^2 + b^2)^{\frac{1}{2}} \left[e^{ax} \cdot (\cos(bx + c + \infty)) + \cos(bx + c + \infty) \cdot \frac{d}{dx} (e^{ax}) \right] \\ &= (a^2 + b^2)^{\frac{1}{2}} \left[e^{ax} \cdot \{ -\sin(bx + c + \infty) \} + \cos(bx + c + \infty) + \cos(bx + c + \infty) \cdot e^{ax} \cdot \frac{d}{dx} (ax) \right] \\ &= (a^2 + b^2)^{\frac{1}{2}} \left[e^{ax} \cdot \{ -\sin(bx + c + \infty) \} \cdot (b \times 1 + 0 + 0) + \cos(bx + c + \infty) \cdot e^{ax} \times a \times 1 \right] \\ &= (a^2 + b^2)^{\frac{1}{2}} \left[-e^{ax} \sin(bx + c + \infty) \times (b \times 1 + 0 + 0) + \cos(bx + c + \infty) \cdot e^{ax} \times a \times 1 \right] \\ &= e^{ax} \cdot (a^2 + b^2)^{\frac{1}{2}} \left[a\cos(bx + c + \infty) - b\sin(bx + c + \infty) \right] \\ &= e^{ax} \cdot (a^2 + b^2)^{\frac{1}{2}} \left[\cos \infty \cdot \cos(bx + c + \infty) - \sin \infty \cdot \sin(bx + c + \infty) \right] \\ &= e^{ax} \cdot (a^2 + b^2)^{\frac{2}{2}} \left[\cos \infty \cdot \cos(bx + c + \infty) - \sin \infty \cdot \sin(bx + c + \infty) \right] \\ &= e^{ax} \cdot (a^2 + b^2)^{\frac{2}{2}} \cdot \cos(bx + c + \infty + \infty) \\ &= e^{ax} \cdot (a^2 + b^2)^{\frac{2}{2}} \cdot \cos(bx + c + 2\infty) \\ \text{Similarly.} \\ &\frac{d^3y}{dx^3} = e^{ax} \cdot (a^2 + b^2)^{\frac{3}{2}} \cdot \cos(bx + c + 3\infty) \\ \text{In general, the n'th order derivative is given by \\ &\frac{d^n y}{dx^n} = e^{ax} \cdot (a^2 + b^2)^{\frac{n}{2}} \cdot \cos(bx + c + n\infty), \\ \text{Where } \infty = \tan^{-1} \left(\frac{b}{a} \right) \\ &\therefore \frac{d^n y}{dx^n} = e^{ax} \cdot (a^2 + b^2)^{\frac{n}{2}} \cdot \cos\left[bx + c + n \tan^{-1} \left(\frac{b}{a} \right) \right]. \end{split}$$

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Exercise 1.5 | Q 4.12 | Page 60

Find the nth derivative of the following : $y = e^{8x} \cdot \cos(6x + 7)$

SOLUTION

$$y = e^{8x} \cdot \cos (6x + 7)$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} [e^{ax} \cdot \cos(6x + 7)]$$

$$= e^{ax} \cdot \frac{d}{dx} [\cos(6x + 7)] + \cos(6x + 7) \cdot \frac{d}{dx} (e^{ax})$$

$$= e^{ax} \cdot [-\sin(6x + 7)] \cdot \frac{d}{dx} (6x + 7) + \cos(6x + 7) \cdot e^{ax} \cdot \frac{d}{dx} (ax)$$

$$= -e^{ax} \sin (6x + 7) \times (b \times 1 + 0) + e^{ax} \cos(6x + 7) \times a \times 1$$

$$= e^{ax} [a \cos (6x + 7) - b \sin (6x + 7)]$$

$$= e^{ax} \cdot \sqrt{a^2 + b^2} \left[\frac{a}{\sqrt{a^2 + b^2}} \cos(6x + 7) - \frac{b}{\sqrt{a^2 + b^2}} \sin(6x + 7) \right]$$

Let $\frac{a}{\sqrt{a^2 + b^2}} = \cos x$ and $\frac{b}{\sqrt{a^2 + b^2}} = \sin x$
Then $\tan \infty = \frac{b}{a}$
 $\therefore \infty = \tan^{-1} \left(\frac{b}{a}\right)$
 $\therefore \frac{dy}{dx} = e^{ax} \cdot \sqrt{a^2 + b^2} [\cos \infty \cdot \cos(bx + c) - \sin \infty \cdot \sin(bx + c)]$

$$= e^{ax} \cdot (a^2 + b^2)^{\frac{1}{2}} \cdot \cos(6x + 7 + x)$$

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 $rac{d^2y}{dx^2} = rac{\mathrm{d}}{\mathrm{dx}} \Big[e^{ax} . \left(a^2+b^2
ight)^{rac{1}{2}} . \cos(6x+7+\infty) \Big]$





$$\begin{split} &= \left(a^{2} + b^{2}\right)^{\frac{1}{2}} \cdot \frac{d}{dx} \left[e^{ax} \cdot \cos(6x + 7 + \infty)\right] \\ &= \left(a^{2} + b^{2}\right)^{\frac{1}{2}} \left[e^{ax} \cdot \left\{\cos(6x + 7 + \infty)\right\} + \cos(6x + 7 + \infty) \cdot \frac{d}{dx}(e^{ax})\right] \\ &= \left(a^{2} + b^{2}\right)^{\frac{1}{2}} \left[e^{ax} \cdot \left\{-\sin(6x + 7 + \infty)\right\} \cdot \frac{d}{dx}(6x + 7 + \infty) + \cos(6x + 7 + \infty) \cdot e^{ax} \cdot \frac{d}{dx}(ax)\right] \\ &= \left(a^{2} + b^{2}\right)^{\frac{1}{2}} \left[-e^{ax}\sin(6x + 7 + \infty) \times (b \times 1 + 0 + 0) + \cos(6x + 7 + \infty) \cdot e^{ax} \times a \times 1\right] \\ &= e^{ax} \cdot \left(a^{2} + b^{2}\right)^{\frac{1}{2}} \left[a\cos(6x + 7 + \infty) - b\sin(6x + 7 + \infty)\right] \\ &= e^{ax} \cdot \left(a^{2} + b^{2}\right)^{\frac{1}{2}} \cdot \sqrt{a^{2} + b^{2}} \left[\frac{a}{\sqrt{a^{2} + b^{2}}}\cos(6x + 7 + \infty) = \frac{b}{\sqrt{a^{2} + b^{2}}}\sin(6x + 7 + \infty)\right] \\ &= e^{ax} \cdot \left(a^{2} + b^{2}\right)^{\frac{1}{2}} \left[\cos \infty \cdot \cos(6x + 7 + \infty) - \sin \infty \cdot \sin(6x + 7 + \infty)\right] \\ &= e^{ax} \cdot \left(a^{2} + b^{2}\right)^{\frac{2}{2}} \cdot \cos(6x + 7 + \infty) - \sin \infty \cdot \sin(6x + 7 + \infty) \\ &= e^{ax} \cdot \left(a^{2} + b^{2}\right)^{\frac{2}{2}} \cdot \cos(6x + 7 + 2\infty) \\ &\text{Similarly.} \\ \frac{d^{3}y}{dx^{3}} &= e^{ax} \cdot \left(a^{2} + b^{2}\right)^{\frac{3}{2}} \cdot \cos(6x + 7 + 3\infty) \\ &\text{In general, the nth order derivative is given by} \\ \frac{d^{n}y}{dx^{n}} &= e^{ax} \cdot \left(a^{2} + b^{2}\right)^{\frac{n}{2}} \cdot \cos(6x + 7 + n\infty), \\ &\text{Where } \infty = \tan^{-1}\left(\frac{b}{a}\right) \\ &\therefore \frac{d^{n}y}{dx^{n}} = e^{8x} \cdot (10)^{n} \cdot \cos\left[6x + 7 + n\tan^{-1}\left(\frac{3}{4}\right)\right]. \end{split}$$

MISCELLANEOUS EXERCISE 1 [PAGES 61 - 63]

Miscellaneous Exercise 1 | Q 1 | Page 61

Choose the correct option from the given alternatives : Let f(1) = 3, $f'(1) = -\frac{1}{3}$, g(1) = -4 and $g'(1) = -\frac{8}{3}$. The derivative of $\sqrt{[f(x)]^2 + [g(x)]^2}$ w.r.t. x at x = 1 is

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$$\begin{array}{r} -\frac{29}{15} \\ \frac{7}{3} \\ \frac{31}{15} \\ \frac{15}{29} \\ 15 \end{array}$$

 $\begin{aligned} \frac{29}{15} \\ \text{[Hint: Let y = } \sqrt{[f(x)]^2 + [g(x)]^2} \\ \text{Then } \frac{\mathrm{dy}}{\mathrm{dx}} &= \frac{1}{\sqrt{[f(x)]^2 + [g(x)]^2}} \cdot [2f(x) \cdot f'(x) + 2g(x) \cdot g'(x)] \\ & \therefore \left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)_{\mathrm{at}x=1} = \frac{1}{\sqrt{[f(x)]^2 + [g(x)]^2}} \cdot [2f(1) \cdot f'(1) + 2g(1) \cdot g'(1)] \\ &= \frac{1}{2\sqrt{9+16}} \times \left[2(3)\left(-\frac{1}{3}\right) + 2(-4)\left(-\frac{8}{3}\right) = \frac{29}{15} \right]. \end{aligned}$

Miscellaneous Exercise 1 | Q 2 | Page 62

Choose the correct option from the given alternatives : If y = sec (tan ⁻¹x), then $\frac{dy}{dx}$ at x = 1, is equal to $\frac{1}{2}$ 1 $\frac{1}{\sqrt{2}}$ $\sqrt{2}$





$$\begin{aligned} \frac{1}{\sqrt{2}} \\ \text{[Hint:} & \frac{\mathrm{dy}}{\mathrm{dx}} = \sec\left(\tan^{-1}x\right) \cdot \tan\left(\tan^{-1}x\right) \times \frac{1}{1+x^2} \\ & \therefore \left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)_{\mathrm{at}x=1} = \sec\left(\tan^{-1}1\right) \times 1 \times \frac{1}{1+1^2} \\ & = \sec\left(\frac{\pi}{4} \times \frac{1}{2}\right) = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}} \end{aligned}$$

Miscellaneous Exercise 1 | Q 3 | Page 62

Choose the correct option from the given alternatives :

If
$$f(x) = \sin^{-1}\left(\frac{4^{x+\frac{1}{2}}}{1+2^{4x}}\right)$$
, which of the following is not the derivative of $f(x)$?

$$\frac{\frac{2.4^{x} \cdot \log 4}{1+4^{2x}}}{\frac{4^{x+1} \cdot \log 2}{1+4^{2x}}}$$

$$\frac{\frac{4^{x+1} \cdot \log 4}{1+4^{4x}}}{\frac{2^{2^{(x+1)} \cdot \log 2}}{1+2^{4x}}}$$

$$\frac{4^{x+1}\log 4}{1+4^{4x}}$$

[Hint : Put
$$4^x$$
 = tan θ . Then θ = tan⁻¹(4^x)

$$\therefore f(x) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan\theta}\right)$$

= sin⁻¹(sin2θ)
= 2θ
= 2tan⁻¹(4^x)
∴ f'(x) = 2 ×
$$\frac{1}{1 + (4^x)^2}$$
 × 4^x log 4
= $\frac{2.4^x \cdot \log 4}{1 + 4^{2x}}$... (a)
= $\frac{2.4^x \cdot 2 \log 2}{1 + 4^{2x}}$
= $\frac{4^{x+1} \cdot \log 2}{1 + 4^{2x}}$...(b)
= $\frac{(2^2)^{x+1} \cdot \log 2}{1 + 2^{4x}}$...(b)

Miscellaneous Exercise 1 | Q 4 | Page 62

Choose the correct option from the given alternatives :

If
$$x^y = y^x$$
, then $\frac{dy}{dx} = \dots$

$$\frac{x(x \log y - y)}{y(y \log x - x)}$$

$$\frac{y(x \log y - y)}{x(y \log x - x)}$$

$$\frac{y^2(1 - \log x)}{x^2(1 - \log y)}$$

$$\frac{y(1 - \log x)}{x(1 - \log y)}$$



$$\frac{y(x \log y - y)}{x(y \log x - x)}$$

$$x^{y} = y^{x} \quad \therefore y \log x = x \log y$$

$$\therefore y \times \frac{1}{x} + (\log x) \frac{dy}{dx} = x \times \frac{1}{y} \frac{dy}{dx} + \log y$$

$$\therefore \left(\log x - \frac{x}{y}\right) \frac{dy}{dx} = \log y - \frac{y}{x}$$

$$\therefore \left(\frac{y \log x - x}{y}\right) \frac{dy}{dx} = \frac{x \log y - y}{x}$$

$$\therefore \left(\frac{y \log x - x}{y}\right) \frac{dy}{dx} = \frac{x \log y - y}{x}$$

Miscellaneous Exercise 1 | Q 5 | Page 62

Choose the correct option from the given alternatives :

If y = sin (2sin⁻¹ x), then dx =

$$\frac{2 - 4x^2}{\sqrt{1 - x^2}}$$

$$\frac{2 + 4x^2}{\sqrt{1 - x^2}}$$

$$\frac{4x^2 - 1}{\sqrt{1 - x^2}}$$

$$\frac{1 - 2x^2}{\sqrt{1 - x^2}}$$





$$\frac{2-4x^2}{\sqrt{1-x^2}}$$
[Hint : y = sin(2sin⁻¹ x)
Put x = sin0. Then0 = sin⁻¹x
 \therefore y = sin20
 $\therefore \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = 2\cos 2\theta \times \frac{1}{\sqrt{1-x^2}}$
 $= \frac{2(1-2\sin^2\theta)}{\sqrt{1-x^2}} = \frac{2-4x^2}{\sqrt{1-x^2}}$].

Miscellaneous Exercise 1 | Q 6 | Page 62

Choose the correct option from the given alternatives :

If
$$y = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right) + \sin\left[2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right] \tan\frac{dy}{dx} = \dots$$

$$\frac{\frac{x}{\sqrt{1-x^2}}}{\frac{1-2x}{\sqrt{1-x^2}}}{\frac{2\sqrt{1-x^2}}{\sqrt{1-x^2}}}$$



$$\begin{aligned} \frac{1-2x}{2\sqrt{1-x^2}} \\ y &= \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right) + \sin\left[2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right] \\ \text{Put } x &= \cos\theta. \text{ Then } \theta = \cos^{-1}x \\ \therefore y &= \tan^{-1}\left(\frac{\cos\theta}{1+\sqrt{1-\cos^2\theta}}\right) + \sin\left[2\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right] \\ &= \tan^{-1}\left(\frac{\cos\theta}{1+\sin\theta}\right) + \sin\left[2\tan^{-1}\sqrt{\frac{2\sin^2\left(\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\theta}{2}\right)}}\right] \\ &= \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{1+\cos\left(\frac{\pi}{2}-\theta\right)}\right] + \sin\left[2\tan^{-1}\left(\tan\left(\frac{\theta}{2}\right)\right)\right] \\ &= \tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\cdot\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}\right] + \sin\left(2\times\frac{\theta}{2}\right) \\ &= \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)+\sin\theta\right] \\ &= \frac{\pi}{4}-\frac{\theta}{2}+\sqrt{1-\cos^2\theta} \\ &= \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x+\sqrt{1-x^2} \\ &\therefore \frac{dy}{dx} = 0 - \frac{1}{2} \times \frac{-1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{1-x^2}} \times (-2x) \end{aligned}$$



$$= \frac{1}{2\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}} \\ = \frac{1-2x}{2\sqrt{1-x^2}}.$$

Miscellaneous Exercise 1 | Q 7 | Page 62

Choose the correct option from the given alternatives :

If y is a function of x and log (x + y) = 2xy, then the value of y'(0) = 2 0

-1

1

SOLUTION

1
[Hint :
$$\log(x + y) = 2xy$$
 ...(1)

$$\therefore \frac{1}{x + y} \cdot \left(1 + \frac{dy}{dx}\right) = 2x \frac{dy}{dx} + 2y$$

$$\therefore \left(\frac{1}{x + y} - 2x\right) \frac{dy}{dx} = 2y - \frac{1}{x + y}$$

$$\therefore \frac{dy}{dx} = \frac{2y(x + y) - 1}{1 - 2x(x + y)}$$
If $x = 0$, then from (1),
 $\log y = 0 = \log 1$
 $\therefore y = 1$
 $\therefore y'(0) = \frac{(2(1)(0 + 1) - 1)}{1 - 2(0)(0 + 1)} = 1$].





Miscellaneous Exercise 1 | Q 8 | Page 62

Choose the correct option from the given alternatives :

If g is the inverse of function f and f'(x) = $\frac{1}{1+x}$, then the value of g'(x) is equal to :

$$\frac{1 + x^{7}}{1 + [g(x)]^{7}}$$

$$\frac{1}{1 + [g(x)]^{7}}$$

$$\frac{1 + [g(x)]^{7}}{7x^{6}}$$

SOLUTION

 $1 + [g(x)]^7$

[Hint : Since g is the inverse of f, $f^{-1}(x) = g(x)$

$$\therefore f[f^{-1}(x)] = f[g(x)] = x$$

$$\therefore f'[g(x)] \cdot \frac{d}{dx}[g(x)] = 1$$

$$\therefore f'[g(x)] \times g'(x) = 1$$

$$\therefore g'(x) = \frac{1}{f'[g(x)]}, \text{ where } f'(x) = \frac{1}{1+x^7}$$

$$\therefore g'(x) = 1 + [g(x)^7].$$

Miscellaneous Exercise 1 | Q 9 | Page 62

Choose the correct option from the given alternatives :

If
$$x\sqrt{y+1} + y\sqrt{x+1} = 0$$
 and $x \neq y$, then $\frac{dy}{dx} = \dots$
$$\frac{1}{(1+x)^2} - \frac{1}{(1+x)^2}$$



$$\frac{(1+x)^2}{x} - \frac{x}{x+1}$$

$$-\frac{1}{(1+x)^2}$$
[Hint: $x\sqrt{y-1} = -y\sqrt{x+1}$
 $\therefore x^2(y+1) = y^2(x+1)$
 $\therefore x^2y + x^2 = xy^2 + y^2$
 $\therefore x^2 - y^2 = xy^2 - x^2y$
 $\therefore (x-y)(x+y) = -xy(x-y)$
 $\therefore x+y = -xy$...[$\because x \neq y$] ...(1)

DIfferentiating both sides w.r.t. x, we get

$$1 + \frac{\mathrm{dy}}{\mathrm{dx}} = -x\frac{\mathrm{dy}}{\mathrm{dx}} - y$$

$$\therefore (1+x)\frac{\mathrm{dy}}{\mathrm{dx}} = -1 - y$$

$$\therefore (1+x)\frac{\mathrm{dy}}{\mathrm{dx}}$$

$$= -(1+x)(1+y)$$

$$= -(1+x+y+xy)$$

$$= -(1-xy+xy) \qquad \dots[By (1)]$$

$$= -1$$

$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = -\frac{1}{(1+x)^2} \right].$$

Miscellaneous Exercise 1 | Q 10 | Page 62

Choose the correct option from the given alternatives :

If
$$y \tan^{-1}\left(\sqrt{\frac{a-x}{a+x}}\right)$$
, where $-a < x < a$, then $\frac{dy}{dx} = \dots$

$$\frac{\frac{x}{\sqrt{a^2 - x^2}}}{\frac{\sqrt{a^2 - x^2}}{2\sqrt{a^2 - x^2}}}$$

$$\frac{1}{2\sqrt{a^2 - x^2}}$$
SOLUTION

$$-\frac{1}{2\sqrt{a^2-x^2}}$$
[Hint : Put x = a cos θ].

Miscellaneous Exercise 1 | Q 11 | Page 63

Choose the correct option from the given alternatives :

If x = a(cos
$$\theta$$
 + θ sin θ), y = a(sin θ - θ cos θ), then $\left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{4}}$ =

$$\frac{8\sqrt{2}}{a\pi} - \frac{8\sqrt{2}}{a\pi} - \frac{a\pi}{a\pi} - \frac{8\sqrt{2}}{a\pi} - \frac{8\sqrt{2$$



 $\frac{8\sqrt{2}}{a\pi}$ $[\mathsf{Hint}:\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)}{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)}$ $=\frac{a(\cos\theta+\theta\sin\theta-\cos\theta)}{a(-\sin\theta+\theta\cos\theta+\sin\theta)}$ $= \frac{a\theta\sin\theta}{a\theta\cos\theta}$ $= tan\theta$ and $rac{d^2 y}{dx^2} = rac{\mathrm{d}}{\mathrm{dx}}(\tan heta)$ $=\frac{\mathrm{d}}{\mathrm{d}\theta}(\tan\theta)\times\frac{\mathrm{d}\theta}{\mathrm{d}\mathbf{x}}$ $= \sec^2 \theta \times \frac{1}{a\theta \cos \theta}$ $=\frac{1}{a^{\Theta}} \cdot \sec^3 \theta$ $\therefore \left(rac{d^2 y}{dx^2}
ight)_{\mathrm{at} heta = rac{\pi}{2}} = rac{1}{a \left(rac{\pi}{4}
ight)} \left(\mathrm{sec} \; rac{\pi}{4}
ight)^3$ $=\frac{4}{2\pi}\times\left(\sqrt{2}\right)^3$ $=\frac{8\sqrt{2}}{a\pi}$.



Miscellaneous Exercise 1 | Q 12 | Page 63

Choose the correct option from the given alternatives :

If
$$y = a \cos(\log x)$$
 and $A \frac{d^2 y}{dx^2} + B \frac{dy}{dx} + C = 0$, then the values of A, B, C are
 $x^2, -x, -y$
 x^2, x, y
 x^2, x, y
SOLUTION
 x^2, x, y
[Hint : $y = a \cos(\log x)$...(1)
 $\therefore \frac{dy}{dx} = a[-\sin(\log x)] \times \frac{1}{x}$
 $\therefore x \frac{dy}{dx^2} + \frac{dy}{dx} = -a \cos(\log x) \times \frac{1}{x}$
 $\therefore x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = -a \cos(\log x) \times \frac{1}{x}$
 $\therefore x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = -y$...[By (1)]
 $\therefore x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$

Comparing this with $A \frac{d^2y}{dx^2} + B \frac{dy}{dx} + C = 0$, we get A = x², B = x, C = y.

MISCELLANEOUS EXERCISE 1 [PAGES 63 - 64]



Miscellaneous Exercise 1 | Q 1 | Page 63

Solve the following :

$$f(x) = -x, \text{ for } -2 \le x < 0$$

= 2x, for $0 \le x < 2$
= $\frac{18 - x}{4}$, for $2 < x \le 7$
g(x) = 6 - 3x, for $0 \le x < 2$
= $\frac{2x - 4}{3}$, for $2 < x \le 7$

Let u(x) = f[g(x)], v(x) = g[f(x)] and w(x) = g[g(x)]. Find each derivative at x = 1, if it exists i.e. find u'(1), v'(1) and w'(1). If it doesn't exist, then explain why?

SOLUTION

$$u(x) = f[g(x)]$$

$$\therefore u'(x) = \frac{d}{dx} \{f[g(x)]\}$$

$$= f'[g(x)] \cdot \frac{d}{dx} [g(x)]$$

$$= f'[gx)] \times g'(x)$$

$$\therefore u'(1) = f'[g(1)] \times g'(1)$$

$$= f'(3) \times g'(1) \qquad \dots(1)$$

$$\dots[\because g(x) = 6 - 3x, 0 \le x \le 2]$$

Now, $f(x) = \frac{18 - x}{4}$, for $2 < x \le 7$
and $g(x) = 6 - 3x$, for $0 < x \le 2$

$$\therefore f'(x) = \frac{1}{4} (0 - 1) = -\frac{1}{4}$$
, for $2 < x \le 7$
and $g'(x) = 0 - 3(1) = -3$, for $0 < x \le 2$

$$\therefore f'(3) = -\frac{1}{4} \text{ and } g'(1) = -3$$

$$\therefore \text{ from (1),}$$

$$u'(1) = -\frac{1}{4} (-3) = \frac{3}{4}$$

Now, v(x) = g[f(x)]
∴ v'(x) =
$$\frac{d}{dx} \{g[f(x)]\}$$

= $g'[f(x)] \cdot \frac{d}{dx} [f(x)]$
= $g'[f(x)] \times f'(x)$
∴ v'(1) = $g'[f(1)] \times f'(x)$
= $g'(2) \times f'(1)$...(2)
...[: $f(x) = 2x, 0 \le x \le 2$]
Now, $g(x) = 6 - 3x$, for $0 \le x \le 2$
= $\frac{2x - 4}{3}$, for $2 < x \le 7$
∴ $g''(x) = 0 - 3 \times 1 = -3$, for $0 \le x \le 2$
and $g'(x) = \frac{1}{3}(2 \times 1 - 0) = \frac{2}{3}$, for $2 < x \le 7$
∴ $Lg'(2) \neq Rg'(2)$
∴ $g'(2)$ does not exist
∴ from (2),
v'(1) does not exist
Also, $w(x) = g[g(x)]$
∴ $w'(x) = \frac{d}{dx} \{g[g(x)]\}$
= $g'[g(x)] \cdot \frac{d}{dx} [g(x)]$
= $g'[g(x)] \times g'(x)$
∴ $w'(1) = g'[g(1)] \times g'(x)$
= $g'(3) \times g'(1)$...(3)
...[: $g(x) = 6 - 3x, 0 \le x \le 2$]
Now, $g(x) = 6 - 3x$, for $0 \le x \le 2$
= $\frac{2x - 4}{3}$, for $2 < x \le 7$



$$\therefore g'(x) = 0 - 3 \times 1 = -3, \text{ for } 0 \le x \le 2$$

and g'(x) = $\frac{1}{3}(2 \times 1 - 0) = \frac{2}{3}, \text{ for } 2 \le x \le 7$
$$\therefore g(3) = \frac{2}{3} \text{ and } g'(1) = -3$$

$$\therefore \text{ from (3),}$$

w'(1) = $\frac{2}{3}(-3) = -2.$
Hence, u'(1) = $\frac{3}{4}$, v'(1) does not exist and w'(1) = -2.

Miscellaneous Exercise 1 | Q 2 | Page 63 Solve the following :

The values of f(x), g(x), f'(x) and g'(x) are given in the following table :

X	f(x)	g(x)	f'(x)	fg'(x)
- 1	3	2	- 3	4
2	2	- 1	- 5	- 4

Match the following:

A Group – Function	B Group – Derivative
${}^{(A)}\frac{\mathrm{d}}{\mathrm{d} \mathrm{x}}[f(g(x))]\mathrm{a} \mathrm{t} x=-1$	1. – 16
(B) $rac{\mathrm{d}}{\mathrm{d} \mathrm{x}}[g(f(x)-1)]\mathrm{at} x=-1$	2. 20
$\text{(C)}\frac{\mathrm{d}}{\mathrm{d}x}[f(f(x)-3)]\mathrm{at}x=2$	3. – 20
$(D)\frac{\mathrm{d}}{\mathrm{d}x}[g(g(x))]\mathrm{a}\mathrm{t}x=2$	5. 12


(A)
$$\frac{d}{dx} [f(g(x))]$$

= $f'(g(x)) \cdot \frac{d}{dx} (g(x))$
= $f'(g(x)) xg'(x)$
 $\therefore \frac{d}{dx} [f(g(x))] at x = -1$
= $f'(g(-1)) x g'(-1)$
= $f'(2) x g'(-1) \dots [\because g(x) = 2, when x = -1]$
= -5×4
= -20
(B) $\frac{d}{dx} [g(f(x) - 1)]$
= $g'(f(x) - 1) \cdot \frac{d}{dx} [f(x) - 1]$
= $g'(f(x) - 1) \cdot [f'(x) - 0]$
 $\therefore \frac{d}{dx} [gf(x) - 1] at x = -1$
= $g'(f(-1) - 1) x f'(-1)$
= $g'(g - 1) x f'(-1) \dots [\because f(x) 33, when x = -1]$
= $g'(2) x f'(-1)$
= $(-4)(-3)$
= 12
(C) $\frac{d}{dx} [f(f(x) - 3)]$
= $f'(f(x) - 3) \cdot \frac{d}{dx} [f(x) - 3]$
= $f'(f(x) - 3) x [f'(x) - 0]$

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$$\therefore \frac{d}{dx} [f(f(x) - 3)] \text{ at } = 2$$

$$= f''(f(2) - 3) \times f'(2)$$

$$= f'(2 - 3) \times f'(2) \quad ...[\because f(x) = 2, \text{ when } x = 2]$$

$$= f'(-1) \times f'(2)$$

$$= (-3)(-5)$$

$$= 15$$
(D) $\frac{d}{dx} [g(g(x))]$

$$= g'(g(x)) \cdot \frac{d}{dx} [g(x)]$$

$$= g'(g(x)) \times g'(x)$$

$$\therefore \frac{d}{dx} [g(g(x))] \text{ at } x = 2$$

$$= g'(g(2)) \times g'(2)$$

$$= g'(-1) c g'(2) \quad ...[\because g(x) = -1 \text{ at } x = 2]$$

$$= 4(-4)$$

$$= -16$$
Hence, (A) $\rightarrow 3$, (B) $\rightarrow 5$, (C) $\rightarrow 4$, (D) $\rightarrow 1$.

Miscellaneous Exercise 1 | Q 3 | Page 63

Suppose that the functions f and g and their derivatives with respect to x have the following values at x = 0 and x = 1:

x	f(x)	g(x)	f')x)	g'(x)
0	1		5	$\frac{1}{3}$
1	3	- 4	$-rac{1}{3}$	$-\frac{8}{3}$





(i) The derivative of f[g(x)] w.r.t. x at x = 0 is (ii) The derivative of g[f(x)] w.r.t. x at x = 0 is (iii) The value of $\left[\frac{d}{dx}\left[x^{10} + f(x)\right]^{-2}\right]_{x=1}$ is (iv) The derivative of f[(x + g(x))] w.r.t. x at x = 0 is ...

SOLUTION

(i)
$$\frac{d}{dx} \{f[g(x)]\}$$

$$= f'[g(x)] \cdot \frac{d}{dx} [g(x)]$$

$$= f'[g(x)] xg'(x)$$

$$\therefore \frac{d}{dx} \{f[g(x)]\} \text{ at } x = 0$$

$$= f'[g(0)] xg'(0)$$

$$= f'(1) xg'(0) \qquad \dots [\because g(x) = 1 \text{ at } x = 0]$$

$$= -\frac{1}{3} \times \frac{1}{3}$$

$$= -\frac{1}{9} \cdot$$
(ii)
$$\frac{d}{dx} \{g[f(x)]\}$$

$$= g'[f(x)] \cdot \frac{d}{dx} [f(x)]$$

$$= g'[f(x)] xf'(x)$$

$$\therefore \frac{d}{dx} \{f[g(x)]\} \text{ at } x = 0$$





$$= g'(1)xf'(0) \qquad ...[\because f(x) = 1 \text{ at } x = 0]$$

$$= -\frac{8}{3} \times 5$$

$$= -\frac{40}{3}.$$
(iii) $\frac{d}{dx} [x^{10} + f(x)]^{-2}$

$$= 2[x^{10} + f(x)]^{-3}. \frac{d}{dx} [x^{10} + f(x)]$$

$$= -2[x^{10} + f(x)]^{-3} \times [10x^9 + f'(x)]$$

$$\therefore \left\{ \frac{d}{dx} [x^{10} f(x)]^{-2} \right\}_{atx=1}$$

$$= -2[1^{10} + f(1)]^{-3} \times [10(1)^9 + f'(1)]$$

$$= \frac{-2}{(1+3)^3} \times \left[10 + \left(-\frac{1}{3}\right) \right] \dots [\because f(x) = 3 \text{ at } x = 1]$$

$$= -\frac{29}{64} \times \frac{29}{3}$$

$$= -\frac{29}{96}.$$
(iv) $\frac{d}{dx} [f(x + g(x))]$

$$= f'(x + g(x)). \frac{d}{dx} [x + g(x)]$$

$$= f'(x + g(x)) \times [1 + g'(x)]$$

$$\therefore \left\{ \frac{d}{dx} [f(x + g(x))] \right\}_{atx=0}$$

$$= f'(0 + g(0))x[1 + g'(0)]$$

$$= f'(1). [1 + g'(0)] \qquad ...[\because g(x) = 1 \text{ at } x = 0]$$



$$= -\frac{1}{3} \left[1 + \frac{1}{3} \right]$$
$$= -\frac{1}{3} \times \frac{4}{3}$$
$$= -\frac{4}{9}.$$

Miscellaneous Exercise 1 | Q 4.1 | Page 64

Differentiate the following w.r.t. x : $\sin\left[2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right]$

SOLUTION

Let y =
$$\sin\left[2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right]$$

Put x = cos
$$\theta$$
. Then θ = cos⁻¹x and
 $\sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$
= $\sqrt{\frac{2\sin^2\left(\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\theta}{2}\right)}}$
= $\sqrt{\tan^2\left(\frac{\theta}{2}\right)}$
= $\tan\left(\frac{\theta}{2}\right)$

$$\therefore \tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\theta}{2}\right)\right]$$

$$= \frac{\theta}{2}$$

$$= \frac{1}{2}\cos^{-1}x$$

$$\therefore y = \sin\left[2 \times \frac{1}{2}\cos^{-1}x\right]$$

$$= \sin\left(\cos^{-1}x\right)$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}[\sin(\cos^{-1}x)]$$

$$= \cos\left(\cos^{-1}x\right) \cdot \frac{d}{dx}(\cos^{-1}x)$$

$$= x \times \frac{-1}{\sqrt{1-x^{2}}}$$

$$= \frac{-x}{\sqrt{1-x^{2}}}.$$

Miscellaneous Exercise 1 | Q 4.2 | Page 64

Differentiate the following w.r.t. x :
$$\sin^2 \left[\cot^{-1} \left(\sqrt{rac{1+x}{1-x}}
ight)
ight]$$

SOLUTION





Let
$$y = \sin^2 \left[\cot^{-1} \left(\sqrt{\frac{1+x}{1-x}} \right) \right]$$

Put $x = \cos\theta$. Then $\theta = \cos^{-1}x$ and
 $\sqrt{\frac{1+x}{1-x}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$
 $= \sqrt{\frac{2\cos^2\left(\frac{\theta}{2}\right)}{2\sin^2\left(\frac{\theta}{2}\right)}}$
 $= \cot\left(\frac{\theta}{2}\right)$
 $\Rightarrow \cot^{-1} \sqrt{\frac{1+x}{1-x}}$
 $= \cot^{-1} \left[\cot\left(\frac{\theta}{2}\right) \right]$
 $= \frac{\theta}{2}$
 $= \frac{1}{2}\cos^{-1}x$
 $\Rightarrow y = \sin^2\left(\frac{1}{2}\cos^{-1}x\right)$
 $\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \left[\sin\left(\frac{1}{2}\cos^{-1}x\right) \right]^2$
 $= 2\sin\left(\frac{1}{2}\cos^{-1}x\right) \cdot \frac{d}{dx}\sin\left(\frac{1}{2}\cos^{-1}x\right)$





$$= 2\sin\left(\frac{1}{2}\cos^{-1}x\right) \cdot \cos\left(\frac{1}{2}\cos^{-1}x\right) \cdot \frac{d}{dx}\left(\frac{1}{2}\cos^{-1}x\right)$$

$$= \sin\left[2\left(\frac{1}{2}\cos^{-1}x\right)\right] \times \frac{1}{2} \cdot \frac{d}{dx}\left(\cos^{-1}x\right)$$

$$= \sin(\cos^{-1}x) \times \frac{1}{2} \times \frac{-1}{\sqrt{1-x^{2}}}$$

$$= \sin\left(\sin^{-1}\sqrt{1-x^{2}}\right) \times \frac{-1}{2\sqrt{1-x^{2}}} \dots \left[\because \cos^{-1}x = \sin^{-1}\sqrt{1-x^{2}}\right]$$

$$= \sqrt{1-x^{2}} \times \frac{-1}{2\sqrt{1-x^{2}}}$$

$$= -\frac{1}{2}.$$

Miscellaneous Exercise 1 | Q 4.3 | Page 64

Differentiate the following w.r.t. x : $an^{-1} igg(rac{\sqrt{x}(3-x)}{1-3x} igg)$

Let
$$y = \tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)$$

 $= \tan^{-1}\left[\frac{3\sqrt{x}-x\sqrt{x}}{1-3x}\right]$
Put $\sqrt{x} = \tan \theta$. Then $\theta = \tan^{-1}(\sqrt{x})$
 $\therefore y = \tan^{-1}\left(\frac{3\tan \theta - \tan^3 \theta}{1-3\tan^2 \theta}\right)$
 $= \tan^{-1}(\tan 3\theta)$
 $= 3\theta$



$$= 3 \tan^{-1}(\sqrt{x})$$

$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = 3 \frac{\mathrm{d}}{\mathrm{dx}} [\tan^{-1}(\sqrt{x})]$$

$$= 3 \times \frac{1}{1 + (\sqrt{x})^2} \cdot \frac{\mathrm{d}}{\mathrm{dx}} (\sqrt{x})$$

$$= \frac{3}{1 + x} \times \frac{1}{2\sqrt{x}}$$

$$= \frac{3}{2\sqrt{x}(1 + x)}.$$

Miscellaneous Exercise 1 | Q 4.4 | Page 64

Differentiate the following w.r.t. x : \cos^{-1}

$$\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{2}\right)$$

SOLUTION

Let y =
$$\cos^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{2}\right)$$

Put
$$x = \cos\theta$$
. Then $\theta = \cos^{-1}x$ and

$$\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{2}\right)$$
$$=\left(\frac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{2}\right)$$
$$=\frac{\sqrt{2\cos^2\left(\frac{\theta}{2}\right)}-\sqrt{2\sin^2\left(\frac{\theta}{2}\right)}}{2}$$

$$= \frac{\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)}{\sqrt{2}}$$

$$= \left[\cos\left(\frac{\theta}{2}\right)\right]\left(\frac{1}{\sqrt{2}}\right) - \left[\sin\left(\frac{\theta}{2}\right)\right]\left(\frac{1}{\sqrt{2}}\right)$$

$$= \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{\pi}{4}\right)$$

$$= \cos\left(\frac{\theta}{2} + \frac{\pi}{4}\right)$$

$$\therefore y = \cos^{-1}\left[\cos\left(\frac{\theta}{2} + \frac{\pi}{4}\right)\right]$$

$$= \frac{\theta}{2} + \frac{\pi}{4}$$

$$= \frac{1}{2}\cos^{-1}x + \frac{\pi}{4}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(\cos^{-1}x) + \frac{d}{dx}\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{2} \times \frac{-1}{\sqrt{1 - x^{2}}} + 0$$

$$= \frac{-1}{2\sqrt{1 - x^{2}}}.$$

Miscellaneous Exercise 1 | Q 4.5 | Page 64

Differentiate the following w.r.t. x :
$$\tan^{-1}\left(\frac{x}{1+6x^2}\right) + \cot^{-1}\left(\frac{1-10x^2}{7x}\right)$$





Let
$$y = \tan^{-1}\left(\frac{x}{1+6x^2}\right) + \cot^{-1}\left(\frac{1-10x^2}{7x}\right)$$

$$= \tan^{-1}\left(\frac{x}{1+6x^2}\right) + \cot^{-1}\left(\frac{7x}{1-10x^2}\right) \dots \left[\because \cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)\right]$$

$$= \tan^{-1}\left[\frac{3x-2x}{1+(3)(2x)}\right] + \tan^{-1}\left[\frac{5x+2x}{1-(5x)(2x)}\right]$$

$$= \tan^{-1}3x - \tan^{-1}2x + \tan^{-1}5x + \tan^{-1}2x$$

$$= \tan^{-1}3x + \tan^{-1}5x$$

$$\therefore \frac{dy}{dx} \left[\tan^{-1}3x + \tan^{-1}5x\right]$$

$$= \frac{d}{dx} (\tan^{-1}3x) + \frac{d}{dx} (\tan^{-1}5x)$$

$$= \frac{1}{1+(3x)^2} \cdot \frac{d}{dx} (3x) + \frac{1}{1+(5x)^2} \cdot \frac{d}{dx} (5x)$$

$$= \frac{1}{1+9x^2} \times 3 \times 1 + \frac{1}{1+25x^2} \times 5 \times 1$$

$$= \frac{3}{1+9x^2} + \frac{5}{1+25x^2}.$$

Miscellaneous Exercise 1 | Q 4.6 | Page 64

Differentiate the following w.r.t. x :
$$an^{-1}\left[\sqrt{rac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}}
ight]$$





Let y =
$$\tan^{-1}\left[\sqrt{\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}}\right]$$

Put x = tan θ . Then θ = tan⁻¹x

$$\begin{split} & \therefore \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x} = \frac{\sqrt{1+\tan^2\theta+\tan\theta}}{\sqrt{1+\tan^2\theta-\tan\theta}} \\ &= \frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta} \\ &= \frac{\left(\frac{1}{\cos\theta}\right)+\left(\frac{\sin\theta}{\cos\theta}\right)}{\left(\frac{1}{\cos\theta}\right)-\left(\frac{\sin\theta}{\cos\theta}\right)} \\ &= \frac{1+\sin\theta}{1-\sin\theta} \\ &= \frac{1-\cos\left(\frac{\pi}{2}+\theta\right)}{1+\cos\left(\frac{\pi}{2}+\theta\right)} \\ &= \frac{2\sin^2\left(\frac{\pi}{4}+\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\pi}{4}+\frac{\theta}{2}\right)} \\ &= \tan^2\left(\frac{\pi}{4}+\frac{\theta}{2}\right) \\ &= \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) \\ &= \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) \end{split}$$





$$\therefore y = \tan^{-1} \left[\tan \left(\frac{\pi}{4} + \frac{\theta}{2} \right) \right]$$
$$= \frac{\pi}{4} + \frac{\theta}{2}$$
$$= \frac{\pi}{4} + \frac{1}{2} \tan^{-1} x$$
$$\therefore \frac{d}{dx} \left(\frac{\pi}{4} \right) + \frac{1}{2} \frac{d}{dx} \left(\tan^{-1} x \right)$$
$$= 0 + \frac{1}{2} \times \frac{1}{1 + x^2}$$
$$= \frac{1}{2(1 + x^2)}.$$

Miscellaneous Exercise 1 | Q 5.1 | Page 64

If
$$\sqrt{y+x} + \sqrt{y-x}$$
 = c, show that $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{y}{x} - \sqrt{\frac{y^2}{x^2}} - 1.$

SOLUTION

$$\sqrt{y+x} + \sqrt{y-x} = c$$

Differentiating both sides w.r.t. x, we get
$$\frac{1}{2\sqrt{y+x}} \cdot \frac{d}{dx}(y+x) + \frac{1}{2\sqrt{y-x}} \cdot \frac{d}{dx}(y-x) = 0$$
$$\therefore \frac{1}{\sqrt{y+x}} \cdot \left(\frac{dy}{dx} + 1\right) + \frac{1}{\sqrt{y-x}} \cdot \left(\frac{dy}{dx} - 1\right) = 0$$
$$\therefore \frac{1}{\sqrt{y+x}} \cdot \frac{dy}{dx} + \frac{1}{\sqrt{y+x}} + \frac{1}{\sqrt{y-x}} \cdot \frac{dy}{dx} - \frac{1}{\sqrt{y-x}} = 0$$
$$\therefore \left(\frac{1}{\sqrt{y+x}} + \frac{1}{\sqrt{y-x}}\right) \frac{dy}{dx} = \frac{1}{\sqrt{y-x}} - \frac{1}{\sqrt{y+x}}$$

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$$\begin{split} & \therefore \left[\frac{\sqrt{y-x} + \sqrt{y+x}}{\sqrt{y+x} \cdot \sqrt{y-x}} \right] \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\sqrt{y+x} + \sqrt{y-x}}{\sqrt{y-x} \cdot \sqrt{y+x}} \\ & \therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\sqrt{y+x} + \sqrt{y-x}}{\sqrt{y+x} \cdot \sqrt{y-x}} \\ & = \frac{\sqrt{y+x} + \sqrt{y-x}}{\sqrt{y+x} + \sqrt{y-x}} \times \frac{\sqrt{y+x} + \sqrt{y-x}}{\sqrt{y+x} - \sqrt{y-x}} \\ & = \frac{\left(\sqrt{y+x} - \sqrt{y-x}^2\right)}{(y+x) - (y-x)} \\ & = \frac{y+x+y-x-2\sqrt{y+x} \cdot \sqrt{y-x}}{y+x-y+x} \\ & = \frac{2y-2\sqrt{y^2-x^2}}{2x} \\ & = \frac{2y}{2x} - \frac{2\sqrt{y^2-x^2}}{2x} \\ & = \frac{2y}{x} - \sqrt{\frac{y^2-x^2}{x^2}} \\ & \therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{y}{x} - \sqrt{\frac{y^2}{x^2} - 1}. \end{split}$$

Miscellaneous Exercise 1 | Q 5.2 | Page 64

If
$$x\sqrt{1-y^2}+y\sqrt{1-x^2}$$
 = 1, then show that $rac{\mathrm{dy}}{\mathrm{dx}}=-\sqrt{rac{1-y^2}{1-x^2}}.$





 $x\sqrt{1-u^2} + u\sqrt{1-x^2} = 1$ $\therefore y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$ Differentiating both sides w.r.t. x, we get $y \cdot \frac{\mathrm{d}}{\mathrm{d}x} \left(\sqrt{1-x^2} \right) + \sqrt{1-x^2} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} + x \cdot \frac{\mathrm{d}}{\mathrm{d}x} \left(\sqrt{1-y^2} \right) + \sqrt{1-y^2} \cdot \frac{\mathrm{d}}{\mathrm{d}x} (x) = 0$ $\therefore y \times \frac{1}{2\sqrt{1-x^2}} \cdot \frac{\mathrm{d}}{\mathrm{dx}} \left(1-x^2\right) + \sqrt{1-x^2} \cdot \frac{\mathrm{dy}}{\mathrm{dx}} + x \times \frac{1}{2\sqrt{1-y^2}} \cdot \frac{\mathrm{d}}{\mathrm{dx}} \left(1-y^2\right) + \sqrt{1-y^2} \times 1 = 0$ $\therefore \frac{y}{2\sqrt{1-x^2}} \times (0-2x) + \sqrt{1-x^2} \cdot \frac{dy}{dx} + \frac{x}{2\sqrt{1-x^2}} \times \left(0-2y\frac{dy}{dx}\right) + \sqrt{1-y^2} = 0$ $\therefore \frac{-xy}{\sqrt{1-x^2}} + \sqrt{1-x^2} \cdot \frac{\mathrm{dy}}{\mathrm{dx}} - \frac{xy}{\sqrt{1-x^2}} \cdot \frac{\mathrm{dy}}{\mathrm{dx}} + \sqrt{1-y^2} = 0$ $\therefore \left(\sqrt{1-x^2} - \frac{\mathrm{xy}}{\sqrt{1-y^2}}\right) \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{xy}}{\sqrt{1-x^2}} - \sqrt{1-y^2}$ $\therefore \left[\frac{\sqrt{1-x^2} \cdot \sqrt{1-y^2} - xy}{\sqrt{1-x^2}} \right] \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{xy - \sqrt{1-x^2} \cdot \sqrt{1-y^2}}{\sqrt{1-x^2}}$ $\therefore \frac{1}{\sqrt{1-x^2}} \cdot \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{-1}{\sqrt{1-x^2}}$ $\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = -\sqrt{\frac{1-y^2}{1-x^2}}.$

Miscellaneous Exercise 1 | Q 5.3 | Page 64

If x sin (a + y) + sin a . cos (a + y) = 0, then show that $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\sin^2(a+y)}{\sin a}$.

SOLUTION

 $x \sin (a + y) + \sin a \cdot \cos (a + y) = 0$...(1)

Differentiating w.r.t. x, we get

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$$\begin{aligned} x \frac{d}{dx} [\sin(a+y)] + \sin(a+y) \cdot \frac{d}{dx}(x) + (\sin a) \cdot \frac{d}{dx} [\cos(a+y)] &= 0 \\ \therefore x \cos(a+y) \cdot \frac{d}{dx}(a+y) + \sin(a+y) \times 1 + (\sin a) [-\sin(a+y)] \cdot \frac{d}{dx}(a+y) &= 0 \\ \therefore x \cos(a+y) \cdot \left(0 + \frac{dy}{dx}\right) + \sin(a+y) - \sin a \cdot \sin(a+y) \left(0 + \frac{dy}{dx}\right) &= 0 \\ \therefore x \cos(a+y) \frac{dy}{dx} + \sin(a+y) - \sin a \cdot \sin(a+y) \frac{dy}{dx} &= 0 \\ \therefore \sin a \cdot \sin(a+y) \frac{dy}{dx} - x \cos(a+y) \frac{dy}{dx} &= \sin(a+y) \\ \therefore [\sin a \cdot \sin(a+y) - x \cos(a+y)] \frac{dy}{dx} &= \sin(a+y) \\ \therefore \frac{dy}{dx} &= \frac{\sin(a+y)}{\sin a \cdot \sin(a+y) - x \cos(a+y)} \\ From (1), \\ x &= \frac{-\sin a \cdot \cos(a+y)}{\sin a \cdot \sin(a+y) + \frac{\sin a \cdot \cos(a+y)}{\sin(a+y)} \cdot \cos(a+y)} \\ &= \frac{\sin^2(a+y)}{\sin a \cdot \sin^2(a+y) + \sin a \cdot \cos^2(a+y)} \\ &= \frac{\sin^2(a+y)}{\sin a [\sin^2(a+y) + \cos^2(a+y)]} \\ \therefore \frac{dy}{dx} &= \frac{\sin^2(a+y)}{\sin a [\sin^2(a+y) + \cos^2(a+y)]} \\ \therefore \frac{dy}{dx} &= \frac{\sin^2(a+y)}{\sin a [\sin^2(a+y) + \cos^2(a+y)]} \end{aligned}$$

Miscellaneous Exercise 1 | Q 5.4 | Page 64

If sin y = x sin (a + y), then show that
$$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\sin^2(a+y)}{\sin a}$$
.



sin y = x sin(a + y)

$$\Rightarrow x = \frac{\sin y}{\sin(a+y)} \qquad \dots (i)$$

Differentiating (i) w.r.t.x,

$$\Rightarrow 1 = \frac{\sin(a+y) \cdot \left(\frac{d}{dx}\sin y\right) - \sin y \cdot \left(\frac{d}{dx}\sin(a+y)\right)}{\sin^2(a+y)}$$
$$\Rightarrow \sin(a+y) \cdot \cos y - \frac{d}{dx} - \sin y \cdot \cos(a+y) \cdot \frac{d}{dx} = \sin^2(a+y)]$$
$$\Rightarrow \frac{d}{dx} [\sin(a+y) \cdot \cos y - \sin y \cdot \cos(a+y)] = \sin^2(a+y)$$
$$\Rightarrow \frac{dy}{dx} [\sin(a+y-y)] = \sin^2(a+y)$$
$$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

Hence proved.

Miscellaneous Exercise 1 | Q 5.5 | Page 64

If
$$x = e^{rac{x}{y}}$$
 , then show that $rac{\mathrm{dy}}{\mathrm{dx}} = rac{x-y}{x\log x}$

SOLUTION

$$x = e^{\frac{x}{y}}$$

$$\therefore \frac{x}{y} = \log x \qquad ...(1)$$

$$\therefore y = \frac{x}{\log x}$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} \left(\frac{x}{\log x}\right)$$



$$= \frac{(\log x) \cdot \frac{d}{dx}(x) - x \cdot \frac{d}{dx}(\log x)}{(\log x)}$$
$$= \frac{(\log x) \times 1 - x \times \frac{1}{x}}{(\log x)^2}$$
$$= \frac{\log x - 1}{(\log x)(\log x)}$$
$$= \frac{\frac{x}{y} - 1}{\left(\frac{x}{y}\right)(\log x)} \qquad \dots [By (1)]$$
$$= \frac{x - y}{x \log x}.$$

Miscellaneous Exercise 1 | Q 5.6 | Page 64

If y = f(x) is a differentiable function of x, then show that $\frac{d^2x}{dy^2} = -\left(\frac{dy}{dx}\right)^{-3} \cdot \frac{d^2y}{dx^2}$.

SOLUTION

If y = f(x) is a differentiable function of x such that inverse function $x = f^{-1}(y)$ exists, then

$$\frac{\mathrm{dx}}{\mathrm{dy}} = \frac{1}{\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)}, \text{where} \frac{\mathrm{dy}}{\mathrm{dx}} \neq 0$$
$$\therefore \frac{\mathrm{d}^2 x}{\mathrm{dy}^2} = \frac{\mathrm{d}}{\mathrm{dy}} \left(\frac{\mathrm{dx}}{\mathrm{dy}}\right)$$
$$= \frac{\mathrm{d}}{\mathrm{dy}} \left[\frac{1}{\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)}\right]$$
$$= \frac{\mathrm{d}}{\mathrm{Dx}} \left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^{-1} \times \frac{\mathrm{dx}}{\mathrm{dy}}$$



$$= -1\left(\frac{dy}{dx}\right)^{-2} \cdot \frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{dy}{dx}\right) \times \frac{1}{\left(\frac{dy}{dx}\right)}$$
$$= -\left(\frac{dy}{dx}\right)^{-2} \cdot \frac{d^2y}{dx^2} \cdot \left(\frac{dy}{dx}\right)^{-1}$$
$$\therefore \frac{d^2x}{dy^2} = -\left(\frac{dy}{dx}\right)^{-3} \cdot \frac{d^2y}{dx^2}.$$

Miscellaneous Exercise 1 | Q 6.1 | Page 64

Differentiate
$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)w.r.t.\tan^{-1}\left(\sqrt{\frac{2x\sqrt{1+x^2}}{1-2x^2}}\right).$$

SOLUTION

Let u =
$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$
 and

v =
$$\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$$
.

Then we want to find $\frac{du}{dv}$

$$\mathsf{u} = \tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right)$$

Put x = tan
$$\theta$$
.
Then θ = tan⁻¹ x
and
 $\frac{\sqrt{1 + +x^2} - 1}{x} = \frac{\sqrt{1 + \tan^2 \theta} - 1}{\tan \theta}$



$$= \frac{\sec \theta - 1}{\tan \theta}$$

$$= \frac{\frac{1}{\cos \theta} - 1}{\left(\frac{\sin \theta}{\cos \theta}\right)}$$

$$= \frac{1 - \cos \theta}{\sin \theta}$$

$$= \frac{2 \sin^2 \left(\frac{\theta}{2}\right)}{2 \sin \left(\frac{\theta}{2}\right) \cos \left(\frac{\theta}{2}\right)}$$

$$= \tan \left(\frac{\theta}{2}\right)$$

$$\therefore u = \tan^{-1} \left[\tan \left(\frac{\theta}{2}\right) \right] = \frac{\theta}{2} = \frac{1}{2} \tan^{-1} x$$

$$\therefore \frac{du}{dx} = \frac{1}{2} \frac{d}{dx} (\tan^{-1} x)$$

$$= \frac{1}{2} \times \frac{1}{1 + x^2}$$

$$= \frac{1}{2(1 + x^2)}$$

$$v = \tan^{-1} \left(\frac{2x\sqrt{1 - x^2}}{1 - 2x^2}\right)$$
Put x = sin θ .





Then
$$\theta = \sin^{-1}x$$

and

$$\frac{2x\sqrt{1-x^{2}}}{1-2x^{2}}$$

$$= \frac{2\sin\theta\sqrt{1-\sin^{2}\theta}}{1-2\sin^{2}\theta}$$

$$= \frac{2\sin\theta\cos\theta}{1-2\sin^{2}\theta}$$

$$= \frac{\sin 2\theta}{\cos 2\theta}$$

$$= \tan 2\theta$$

$$\therefore v = \tan^{-1}(\tan 2\theta)$$

$$= 2\theta$$

$$= 2\sin^{-1}x$$

$$\therefore \frac{dv}{dx} = 2\frac{d}{dx}(\sin^{-1}x)$$

$$= 2 \times \frac{1}{\sqrt{1-x^{2}}} = \frac{2}{\sqrt{1-x^{2}}}$$

$$\therefore \frac{dv}{dx} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)}$$

$$= \frac{\left[\frac{1}{2(1+x^{2})}\right]}{\left(\frac{2}{\sqrt{1-x^{2}}}\right)}$$

$$= \frac{1}{2(1+x^{2})} \times \frac{\sqrt{1-x^{2}}}{2}$$

$$= \frac{\sqrt{1-x^{2}}}{4(1+x^{2})}.$$



Miscellaneous Exercise 1 | Q 6.2 | Page 64

Differentiate log
$$\left[\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x} \right]$$
 w.r.t. cos (log x).

SOLUTION

Let y = log
$$\left[\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right]$$
 and v = cos (log x)

Then we want to find $\frac{du}{dv}$.

$$u = \log\left(\frac{\sqrt{1+x^{2}+x}}{\sqrt{1+x^{2}+x}} \times \frac{\sqrt{1+x^{2}+x}}{\sqrt{1+x^{2}+x}}\right)$$

= $\log\left[\frac{\left(\sqrt{1+x^{2}+x}\right)^{2}}{1+x^{2}-x^{2}}\right]$
= $2\log\left(\sqrt{1+x^{2}}+x\right)$
 $\therefore \frac{du}{dx} = 2\frac{d}{dx}\left[\log\left(\sqrt{1+x^{2}}+x\right)\right]$
= $\frac{2}{\sqrt{1+x^{2}+x}} \cdot \frac{d}{dx}\left(\sqrt{1+x^{2}}+x\right)$
= $\frac{2}{\sqrt{1+x^{2}+x}} \cdot \left[\frac{1}{2\sqrt{1+x^{2}}} \cdot \frac{d}{dx}(1+x^{2})+1\right]$





$$= \frac{2}{\sqrt{1+x^2+x}} \cdot \left[\frac{2x}{2\sqrt{1+x^2}} + 1\right]$$

$$= \frac{2}{\sqrt{1+x^2+x}} \left(\frac{x}{\sqrt{1+x^2}} + 1\right)$$

$$= \frac{2\left(x+\sqrt{1+x^2}\right)}{\left(\sqrt{1+x^2+x}\right)\sqrt{1+x^2}}$$

$$= \frac{2}{\sqrt{1+x^2}}$$

$$\frac{dv}{dx} = \frac{d}{dx} [\cos(\log x)]$$

$$= -\sin(\log x) \frac{d}{dx} (\log x)$$

$$= [-\sin(\log x)] \times \frac{1}{x}$$

$$= \frac{-\sin(\log x)}{x}$$

$$\therefore \frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dv}\right)}$$

$$\therefore \frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dv}\right)}$$

$$= \frac{\left(\frac{2}{\sqrt{1+x^2}}\right)}{\left[\frac{(-\sin(\log x))}{x}\right]}$$

Miscellaneous Exercise 1 | Q 6.3 | Page 64

Differentiate
$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)w.r.t.\cos^{-1}\left(\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}\right).$$

SOLUTION

Let
$$u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$
 and $v = \cos^{-1}\left(\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}\right)$

Then we want to find —

Put s = tan
$$\theta$$
. Then θ = tan⁻¹x.
 $\sqrt{1 + x^2} - 1$

Also,
$$\frac{\sqrt{1 + x^2 - 1}}{x}$$
$$= \frac{\sqrt{1 + \tan^2 \theta - 1}}{\tan \theta}$$
$$= \frac{\sec \theta - 1}{\tan \theta}$$
$$= \frac{\frac{1}{\cos \theta} - 1}{\left(\frac{\sin \theta}{\cos \theta}\right)}$$
$$= \frac{1 - \cos \theta}{\sin \theta}$$





$$= \frac{2\sin^2\left(\frac{\theta}{2}\right)}{2\sin\left(\frac{\theta}{2}\right) \cdot \cos\left(\frac{\theta}{2}\right)}$$

$$= \tan\left(\frac{\theta}{2}\right)$$

and

$$\frac{1 + \sqrt{1 + x^2}}{2\sqrt{1 + x^2}}$$

$$= \frac{1 + \sqrt{1 + \tan^2 \theta}}{2\sqrt{1 + \tan^2 \theta}}$$

$$= \frac{1 + \sec \theta}{2 \sec \theta}$$

$$= \frac{1 + \frac{1}{\cos \theta}}{\left(\frac{2}{\cos \theta}\right)}$$

$$= \frac{1 + \cos \theta}{2}$$

$$= \frac{2\cos^2\left(\frac{\theta}{2}\right)}{2}$$

$$= \cos^2\left(\frac{\theta}{2}\right)$$

$$\therefore \sqrt{\frac{1 + \sqrt{1 + x^2}}{2\sqrt{1 + x^2}}} = \cos\left(\frac{\theta}{2}\right)$$

$$\therefore u = \tan^{-1}\left[\tan\left(\frac{\theta}{2}\right)\right] \text{ and } v = \cos^{-1}\left[\cos\left(\frac{\theta}{2}\right)\right)\right]$$

$$\therefore u = \left(\frac{\theta}{2}\right) \text{ and } v = \left(\frac{\theta}{2}\right)$$





$$\therefore$$
 u = $rac{1}{2} an^{-1}x$ and $v=rac{1}{2} an^{-1}x$

Differentiating u and v w.r.t. x, we get

$$\frac{\mathrm{du}}{\mathrm{dx}} = \frac{1}{2} \frac{\mathrm{d}}{\mathrm{dx}} (\tan^{-1} x)$$

$$= \frac{1}{2} \times \frac{1}{1+x^{2}}$$

$$= \frac{1}{2(1+x^{2})}$$
and
$$\frac{\mathrm{dv}}{\mathrm{dx}} = \frac{1}{2} \frac{\mathrm{d}}{\mathrm{dx}} (\tan^{-1} x)$$

$$= \frac{1}{2} \times \frac{1}{1+x^{2}}$$

$$= \frac{1}{2(1+x^{2})}$$

$$\therefore \frac{\mathrm{du}}{\mathrm{dv}} = \frac{\left(\frac{\mathrm{du}}{\mathrm{dx}}\right)}{\left(\frac{\mathrm{dv}}{\mathrm{dx}}\right)}$$

$$= \frac{\frac{1}{2(1+x^{2})}}{\frac{1}{2(1+x^{2})}} = 1.$$
Remark : $u = \frac{1}{2} \tan^{-1} x$ and $v = \frac{1}{2} \tan^{-1} x$
 $\therefore u = v$

$$\therefore \frac{\mathrm{du}}{\mathrm{dv}} = \frac{\mathrm{d}}{\mathrm{dv}} (v) = 1.$$

Miscellaneous Exercise 1 | Q 7.1 | Page 64

If
$$y^2 = a^2 \cos^2 x + b^2 \sin^2 x$$
, show that $y + \frac{d^2 y}{dx^2} = \frac{a^2 b^2}{y^3}$

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$$y^2 = a^2 \cos^2 x + b^2 \sin^2 x$$
 ...(1)

Differentiating both sides w.r.t. x, we get

$$2y\frac{\mathrm{d}y}{\mathrm{d}x} = a^2 \frac{\mathrm{d}}{\mathrm{d}x} (\cos x)^2 + b^2 \frac{\mathrm{d}}{\mathrm{d}x} (\sin x)^2$$
$$= a^2 \times 2\cos x. \frac{\mathrm{d}}{\mathrm{d}x} (\cos x) + b^2 \times 2\sin x. \frac{\mathrm{d}}{\iota} dx (\sin x)$$
$$= a^2 \times 2\cos x (-\sin x) + b^2 \times 2\sin x \cos x$$
$$= (b^2 - a^2)\sin 2x$$
$$\therefore y\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{b^2 - a^2}{2}\right)\sin 2x \quad \dots (2)$$

Differentiating again w.r.t. x, we get

$$y \cdot \frac{d}{dx} \left(\frac{dy}{dx}\right) + \frac{dy}{dx} \cdot \frac{dy}{dx} = \left(\frac{b^2 - a^2}{2}\right) \cdot \frac{d}{dx} (\sin 2x)$$
$$\therefore y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \left(\frac{b^2 - a^2}{2}\right) \times \cos 2x \times 2$$
$$\therefore y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = (b^2 - a^2) \cos 2x$$
$$\therefore y^3 \frac{d^2 y}{dx^2} + y^2 \left(\frac{dy}{dx}\right)^2 = y^2 (b^2 - a^2) \cos 2x$$
$$\therefore y^3 \frac{d^2 y}{dx^2} = y^2 (b^2 - a^2) \cos 2x - y^2 \left(\frac{dy}{dx}\right)^2$$
$$\therefore y^4 + y^3 \frac{d^2 y}{dx^2} = y^2 (b^2 - a^2) \cos 2x - y^2 \left(\frac{dy}{dx}\right)^2 + y^4$$

= $(a^2\cos^2 x + b^2\sin^2 x)(b^2 - a^2)(\cos^2 x - \sin^2 x) - [(b^2 - a^2)\sin x \cos x]^2 + (a^2\cos^2 x + b^2\sin^2 x)^2$...[By (1) and (2)]





$$= (a^{2}b^{2}\cos^{2}x - a^{4}\cos^{2}x + b^{4}\sin^{2}x - a^{2}b^{2}\sin^{2}x) \times (\cos^{2}x - \sin^{2}x) - (b^{4}\sin^{2}x\cos^{2}x + a^{4}\sin^{2}x\cos^{2}x - 2a^{2}b^{2}\sin^{2}x\cos^{2}x) + (a^{4}\cos^{4}x + b^{4}\sin^{4}x + b^{4}\sin^{4}x + 2a^{2}b^{2}\sin^{2}x\cos^{2}x)$$

 $= a^{2}b^{2}\cos 4x - a^{2}b^{2}\sin^{2}x\cos^{2}x - a^{4}\cos^{4}x + a^{4}\sin^{2}x\cos^{2}x + b^{4}\sin^{2}x\cos^{2}x - b^{4}\sin^{2}x\cos^{2}x - a^{4}\sin^{2}x\cos^{2}x + 2a^{2}b^{2}\sin^{2}x\cos^{2}x + a^{4}\cos^{4}x + b^{4}x + b^{4}\sin^{4}x + 2a^{2}b^{2}\sin^{2}x\cos^{2}x$

 $= a^{2}b^{2}\cos^{4}x + 2a^{2}b^{2}\sin^{2}x\cos^{2}x + a^{2}b^{2}\sin^{4}x$ $= a^{2}b^{2} (\sin^{4}x + 2\sin^{2}x\cos^{2}x + \cos^{4}x)$

$$\therefore y^4 + y^3 \frac{d^2 y}{dx^2} = a^2 b^2 \qquad \dots [\because \sin^2 x + \cos^2 x = 1]$$
$$\therefore y^3 \left(y + \frac{d^2 y}{dx^2} \right) = a^2 b^2$$
$$\therefore y + \frac{d^2 y}{dx^2} = \frac{a^2 b^2}{y^3}.$$

Miscellaneous Exercise 1 | Q 7.2 | Page 64

If log y = log (sin x) – x², show that
$$\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (4x^2 + 3)y = 0.$$

SOLUTION

$$\log y = \log (\sin x) - x^{2}$$

$$\therefore \log y = \log(\sin x) - \log e^{x^{2}}$$

$$\therefore \log y = \log\left(\frac{\sin x}{e^{x^{2}}}\right)$$

$$\therefore y = \frac{\sin x}{e^{x^{2}}}$$

$$\therefore e^{x^{2}} \cdot y = \sin x \qquad ...(1)$$

Differentiating both sides w.r.t. x, we get

$$e^{x^{2}} \cdot \frac{\mathrm{dy}}{\mathrm{dx}} + y \cdot \frac{\mathrm{d}}{\mathrm{dx}} e^{x^{2}} = \frac{\mathrm{d}}{\mathrm{dx}} (\sin x)$$
$$\therefore e^{x^{2}} \cdot \frac{\mathrm{dy}}{\mathrm{dx}} + y \cdot e^{x^{2}} \cdot \frac{\mathrm{d}}{\mathrm{dx}} (x^{2}) = \cos x$$





$$\therefore e^{x^2} \cdot \frac{\mathrm{dy}}{\mathrm{dx}} + y \cdot e^{x^2} \times 2x = \cos x$$
$$\therefore e^{x^2} \left(\frac{\mathrm{dy}}{\mathrm{dx}} + 2xy \right) = \cos x$$

DIfferentiating again w.r.t. x, we get

$$e^{x^{2}} \cdot \frac{d}{dx} \left(\frac{dy}{dx} + 2xy \right) + \left(\frac{dy}{dx} + 2xy \right) \cdot \frac{d}{dx} \left(e^{x^{2}} \right) = \frac{d}{dx} (\cos x)$$

$$\therefore e^{x^{2}} \left[\frac{d^{2}y}{dx^{2}} + 2\left(x\frac{dy}{dx} + y \times 1 \right) \right] + \left(\frac{dy}{dx} + 2xy \right) \cdot e^{x^{2}} \cdot \frac{d}{dx} \left(x^{2} \right) = -\sin x$$

$$\therefore e^{x^{2}} \left[\frac{d^{2}y}{dx^{2}} + 2x\frac{dy}{dx} + y \right] + \left(\frac{dy}{dx} + 2xy \right) \cdot e^{x^{2}} \times 2x = -\sin x$$

$$\therefore e^{x^{2}} \left[\frac{d^{2}y}{dx^{2}} + 2x\frac{dy}{dx} + 2y + 2x\frac{dy}{dx} + 4x^{2}y \right]$$

$$= -e^{x^{2}} \cdot y \qquad \dots [By (1)]$$

$$\therefore \frac{d^{2}y}{dx^{2}} + 4x\frac{dy}{dx} + 4x^{2}y + y = -y$$

$$\therefore \frac{d^{2}y}{dx^{2}} + 4x\frac{dy}{dx} + (4x^{2} + 3)y = 0.$$

Miscellaneous Exercise 1 | Q 7.3 | Page 64

If x= a cos
$$\theta$$
, y = b sin θ , show that $a^2 \left[y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 \right] + b^2 = 0.$

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SOLUTION

x= a cos
$$\theta$$
, y = b sin θ
Differentiating x and y w.r.t. θ , we get
 $\frac{dx}{d\theta} = a \frac{d}{d\theta} (\cos \theta) = a(-\sin \theta) = -\sin \theta$...(1)
and

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$$\begin{split} \frac{\mathrm{d}y}{\mathrm{d}\theta} &= \frac{\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)}{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)} \\ &= \frac{b\cos\theta}{-a\sin\theta} \\ &= \left(-\frac{b}{a}\right)\cot\theta \\ &\therefore \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}x} \left[\left(-\frac{b}{a}\right)\cot\theta\right] \\ &= \left(-\frac{b}{a}\right) \cdot \frac{\mathrm{d}}{\mathrm{d}\theta}(\cot\theta) \cdot \frac{\mathrm{d}\theta}{\mathrm{d}x} \\ &= \left(-\frac{b}{a}\right) \cdot \left(-\csc^2\theta\right) \times \frac{1}{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)} \\ &= \left(-\frac{b}{a}\right)\csc^2\theta \times \frac{1}{-a\sin\theta} \quad \dots [\mathrm{By}\ (1)] \\ &= \left(-\frac{b}{a^2}\right)\csc^2\theta \\ &\therefore a^2 \left[y\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right] + b^2 \\ &= a^2 \left[b\sin\theta \cdot \left(-\frac{b}{a^2}\right)\csc^3\theta + \left\{\left(-\frac{b}{a}\right)\cot\theta\right\}^2\right] + b^2 \\ &= a^2 \left[-\frac{b^2}{a^2}\csc^2\theta + \frac{b^2}{a^2}\cot^2\theta\right] + b^2 \\ &= a^2 \left(-\frac{b^2}{a^2}\right)(\csc^2\theta - \cot^2\theta) + b^2 \end{split}$$

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$$= -b^{2} + b^{2} \qquad \dots [\because \operatorname{cosec}^{2}\theta - \operatorname{cot}^{2}\theta = 1]$$
$$\therefore a^{2} \left[y \frac{d^{2}y}{dx^{2}} + \left(\frac{dy}{dx} \right)^{2} \right] + b^{2} = 0.$$

Miscellaneous Exercise 1 | Q 7.4 | Page 64 If $y = A \cos (\log x) + B \sin (\log x)$, show that $x^2y^2 + xy_1 + y = 0$.

SOLUTION

$$y = A \cos (\log x) + B \sin (\log x) \qquad ...(1)$$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = A \frac{d}{dx} [\cos(\log x)] + B \frac{d}{dx} [\sin(\log x)]$$

$$= A [-\sin(\log x)] \cdot \frac{d}{dx} (\log x) + B \cos(\log x) \cdot \frac{d}{dx} (\log x)$$

$$= A \sin(\log x) \times \frac{1}{x} B \cos(\log x) \times \frac{1}{x}$$

$$\therefore x \frac{d}{dx} \left(\frac{dy}{dx}\right) + \frac{dy}{dx} \cdot \frac{d}{dx} (x) = -A \frac{d}{dx} [\sin(\log x)] + B \frac{d}{dx} [\cos(\log x)]$$

$$\therefore x \frac{d^2 y}{dx^2} + \frac{dy}{dx} \times 1 = -A \cos(\log x) \cdot \frac{d}{dx} (\log x) + B[-\sin(\log x)] \cdot \frac{d}{dx} (\log x)$$

$$\therefore xy_2 + y_1 = -A \cos(\log x) \times \frac{1}{x} - B \sin(\log x) \times \frac{1}{x}$$

$$\therefore x^2y_2 + xy_1 = -[A \cos(\log x) + B \sin(\log x)] \dots [By (1)]$$

$$\therefore x^2y_2 + xy_1 + y = 0.$$

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Miscellaneous Exercise 1 | Q 7.5 | Page 64

If $y = Ae^{mx} + Be^{nx}$, show that $y_2 - (m + n)y_1 + mny = 0$.

 $y = Ae^{mx} + Be^{nx}$ Differentiating w.r.t. x, we get $\frac{\mathrm{d}\mathbf{y}}{\mathrm{d}\mathbf{x}} = \mathbf{A}\frac{\mathrm{d}}{\mathrm{d}\mathbf{x}}(e^{mx}) + \mathbf{B}\frac{\mathrm{d}}{\mathrm{d}\mathbf{x}}(e^{mx})$ = Ae^{mx}. $\frac{\mathrm{d}}{\mathrm{dx}}(mx) + Be^{nx}. \frac{\mathrm{d}}{\mathrm{dx}}(nx)$ = Ae^{mx}.m + Be^{nx}.n $= y_1 = mAe^{mx} + nBe^{nx}$...(2) Differentiating again w.r.t. x, we get $y_2 = mA \frac{d}{dx}(e^{mx}) + nB \frac{d}{dx}(e^{nx})$ = $m \operatorname{Ae}^{mx}$. $\frac{\mathrm{d}}{\mathrm{dx}}(mx) + n \operatorname{Be}^{nx}$. $\frac{\mathrm{d}}{\mathrm{dx}}(nx)$ = mAe^{mx}.m + nBe^{nx}.n $\therefore y_2 = m^2 A e^{mx} + n^2 B e^{nx}$...(3) $\therefore y_2 - (m + n)y_1 + mny = (m^2Ae^{mx} + n^2Be^{nx}) - (m + n)(mAe^{mx} + nBe^{nx}) + mn(Ae^{mx} + Be^{nx}) \quad ...[By (1), (2), (3)]$ $= m^{2}Ae^{mx} + n^{2}Be^{nx} - m^{2}Ae^{mx} - mnBe^{mx} - n^{2}Be^{nx} + mnAe^{mx} + mnBe^{nx}$ = 0 $\therefore y_2 - (m + n)y_1 + mny = 0.$



